5. Pipe Flow

5.1 General. Pipe flow exists when a closed conduit of any form is flowing full of water. In pipe flow the cross-sectional area of flow is fixed by the cross section of the conduit and the water surface is not exposed to the atmosphere. The internal pressure in a pipe may be equal to, greater than, or less than the local atmospheric pressure.

The principles of pipe flow apply to the hydraulics of such structures as culverts, drop inlets, regular and inverted siphons, and various types of pipe lines.

If pipe flow exists for a range of discharges under study, the cross-sectional area of flow and the hydraulic radius remain constant for any particular cross section and the velocity is directly proportional to the discharge.

5.2 Fundamentals. The following discussions under subsection 3, Fundamentals of Water Flow, should be reviewed: 3.1, Laminar and Turbulent Flow; 3.2, Continuity of Flow; 3.3, Energy and Head; 3.4, Bernoulli Theorem; and 3.5, Hydraulic Gradient and Energy Gradient.

As pointed out in paragraphs 3.1, the possibility of laminar flow in pipes when Reynold's number exceeds 3000 is very remote. For our purposes this value will be assumed to be the lower limit for turbulent flow.

Reynold's number for pipes is:

$$R = \frac{vd}{\nu} \qquad (5.5\text{-}1)$$

where

R = Reynold's number
v = mean velocity in pipe in ft. per sec.
d = diameter of pipe in feet
ν = (nu) kinematic viscosity in ft.^{2} per sec.
d_i = diameter of pipe in inches.

The kinematic viscosity and other physical properties of water are given in table 5.5-1 for atmospheric pressure. The kinematic viscosity is not affected appreciably by the variation in pressures that can normally be expected in our work.

For example: Based on the above assumption of a minimum value of $R = 3000$, compute the minimum permissible velocity in a 2-inch diameter pipe at which water with a temperature of 50° F. will flow turbulently.

$$v = \frac{R\nu}{d} = \frac{3000 \times 1.41 \times 10^{-5}}{\left(\frac{2}{12}\right)} = 0.254 \text{ fps}$$

TABLE 5.5-1

Temperature Degrees Fahrenheit	Specific Weight lbs./ft.^{3}	Kinematic Viscosity ft.^{2}/sec.	Vapor Pressure psi
32	62.4	1.93×10^{-5}	0.08
40	62.4	1.67×10^{-5}	0.11
50	62.4	1.41×10^{-5}	0.17
60	62.4	1.21×10^{-5}	0.26
70	62.3	1.05×10^{-5}	0.36
80	62.2	0.93×10^{-5}	0.51
90	62.1	0.82×10^{-5}	0.70
100	62.0	0.74×10^{-5}	0.96

To assure turbulent flow, the value of R should be equal to or greater than 3000. Assuming a value of kinematic viscosity, ν , of 0.0000105 for water at 70° F., substituting ($Q \div a$) for v, $0.7854d^2$ for a, and reducing d to d_i gives a minimum discharge for turbulent flow in terms of the diameter of the pipe.

$$Q_{min.} = 2.0617 \times 10^{-3} \; d_i \qquad (5.5\text{-}2)$$

5.3 Friction Loss. The loss of energy or head resulting from turbulence created at the boundary between the sides of the conduit and the flowing water is called friction loss.

In a straight length of conduit, flowing full, with constant cross section and uniform roughness, the rate of loss of head by friction is constant and the energy gradient has a slope, in the direction of flow, equal to the friction head loss per foot of conduit.

Of the many equations that have been developed to express friction loss, the following two equations have been selected for inclusion herein; they are widely used and reliable.

5.3.1 Manning's Formula. The general form of this equation is:

$$v = \frac{1.486}{n} r^{2/3} \, s^{1/2} \qquad (5.5\text{-}3)$$

Nomenclature:

- a = cross-sectional area of flow in ft.^{2}
- d = diameter of pipe in feet.
- d_i = diameter of pipe in inches.
- g = acceleration of gravity = 32.2 ft. per sec.^{2}
- H_l = loss of head in feet due to friction in length, L.
- K_c = head loss coefficient for any conduit.
- K_p = head loss coefficient for circular pipe.
- L = length of conduit in feet.
- n = Manning's roughness coefficient.

p = wetted perimeter in feet.
r = hydraulic radius in feet = $(a \div p) = (d \div 4)$ for round pipe.
s = loss of head in feet per foot of conduit = slope of energy grade and hydraulic grade lines in straight conduits of uniform cross section = $(H_1 \div L)$.
v = mean velocity of flow in ft. per sec.
Q = discharge or capacity in ft.^{3} per sec.

Starting with equation (5.5-3) solve for s, multiply numerator and denominator of right side of equation by 2g, and substitute $(H_1 \div L)$ for s. The result is:

$$H_1 = K_c L \frac{v^2}{2g} \tag{5.5-4}$$

where

$$K_c = \frac{29.164\ n^2}{r^{4/3}} \tag{5.5-5}$$

Adaption of this equation (5.5-5) to circular pipes involves the substitution of $(d \div 4)$ for r and the change from d to d_i.

$$K_p = \frac{5087\ n^2}{d_i^{\,4/3}} \tag{5.5-6}$$

Tables of values for K_p and K_c for the usual ranges of variables encountered are given in drawing ES-42. The 1/3, 2/3, 3/2, 3/4, and 4/3 powers of numbers frequently used in Manning's formula can be found from drawing ES-37.

In some cases it is useful to consider the conditions of flow in a straight conduit of uniform cross section and roughness coefficient when the conduit is on neutral slope. Neutral slope is defined as that slope of a conduit at which the friction loss per foot $(H_1 \div L)$ is equal to the slope of the conduit, i.e., when the conduit is parallel to the hydraulic gradient and energy gradient. In a conduit of given cross section, roughness coefficient and slope, this condition occurs for only one discharge. In figure 5.5-1 the conduit is on neutral slope when

$$\sin \theta = \frac{H_1}{L} = K_c \frac{v^2}{2g} \tag{5.5-7}$$

and

$$s_n \text{ (neutral slope)} = \tan \theta = \frac{K_c \frac{v^2}{2g}}{\sqrt{1 - \left(K_c \frac{v^2}{2g}\right)^2}} \tag{5.5-8}$$

In any case of pipe flow in a conduit of uniform cross section, the loss of head per foot of conduit is:

$$s = K_c \frac{v^2}{2g} \tag{5.5-9}$$

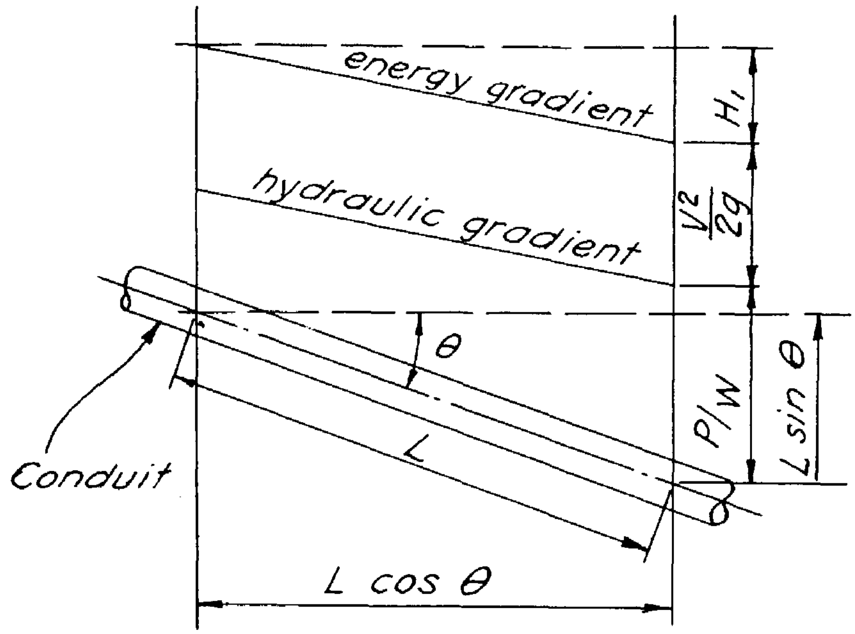


FIG. 5.5-1

The total head loss in a given length of conduit can be determined from equation (5.5-4) with the proper value of K_p or K_c selected from drawing ES-42.

King's Handbook, pp. 188 and 189, gives a number of convenient working forms of Manning's formula and references to tables that will facilitate their use. Four of these are:

$$H_1 = 2.87 \, n^2 \, \frac{Lv^2}{d^{4/3}} \tag{5.5-10}$$

$$H_1 = 4.66 \, n^2 \, \frac{LQ^2}{d^{16/3}} \tag{5.5-11}$$

$$d = \left(\frac{2.159 \, Qn}{s^{1/2}}\right)^{3/8} \tag{5.5-12}$$

$$d_i = \left(\frac{1630 \, Qn}{s^{1/2}}\right)^{3/8} \tag{5.5-13}$$

Drawing ES-54, which is based on equation (5.5-13), may be used to determine d_i, s, or Q, when two of these quantities and n are known.

5.3.2 Hazen-Williams Formula. As generally used, this formula is:

$$v = 1.318 \, C \, r^{0.63} \, s^{0.54} \tag{5.5-14}$$

Notation is the same as given in subsection 5.3.1 with the addition of C, the coefficient of roughness in Hazen-Williams formula.

HYDRAULICS: HEAD LOSS COEFFICIENTS FOR CIRCULAR AND SQUARE CONDUITS FLOWING FULL

HEAD LOSS COEFFICIENT, K_p, FOR CIRCULAR PIPE FLOWING FULL $K_p = \frac{5087\,n^2}{d_i^{4/3}}$

Pipe diam. inches	Flow area sq. ft.	MANNING'S COEFFICIENT OF ROUGHNESS "n"															
		0.010	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.018	0.019	0.020	0.021	0.022	0.023	0.024	0.025
6	0.196	0.0467	0.0565	0.0672	0.0789	0.0914	0.1050	0.1194	0.1348	0.151	0.168	0.187	0.206	0.226	0.247	0.269	0.292
8	0.349	.0318	.0385	.0458	.0537	.0623	.0715	.0814	.0919	.1030	.1148	.1272	.140	.154	.168	.183	.199
10	0.545	.0236	.0286	.0340	.0399	.0463	.0531	.0604	.0682	.0765	.0852	.0944	.1041	.1143	.1249	.136	.148
12	0.785	.0185	.0224	.0267	.0313	.0363	.0417	.0474	.0535	.0600	.0668	.0741	.0817	.0896	.0980	.1067	.1157
14	1.069	.0151	.0182	.0217	.0255	.0295	.0339	.0386	.0436	.0488	.0544	.0603	.0665	.0730	.0798	.0868	.0942
15	1.23	.0138	.0166	.0198	.0232	.0270	.0309	.0352	.0397	.0446	.0496	.0550	.0606	.0666	.0727	.0792	.0859
16	1.40	.0126	.0153	.0182	.0213	.0247	.0284	.0323	.0365	.0409	.0455	.0505	.0556	.0611	.0667	.0727	.0789
18	1.77	.01078	.0130	.0155	.0182	.0211	.0243	.0276	.0312	.0349	.0389	.0431	.0476	.0522	.0570	.0621	.0674
21	2.41	.00878	.01062	.0126	.0148	.0172	.0198	.0225	.0254	.0284	.0317	.0351	.0387	.0425	.0464	.0506	.0549
24	3.14	.00735	.00889	.01058	.0124	.0144	.0165	.0188	.0212	.0238	.0265	.0294	.0324	.0356	.0389	.0423	.0459
27	3.98	.00628	.00760	.00904	.01061	.0123	.0141	.0161	.0181	.0203	.0227	.0251	.0277	.0304	.0332	.0362	.0393
30	4.91	.00546	.00660	.00786	.00922	.01070	.01228	.0140	.0158	.0177	.0197	.0218	.0241	.0264	.0289	.0314	.0341
36	7.07	.00428	.00518	.00616	.00723	.00839	.00963	.01096	.0124	.0139	.0154	.0171	.0189	.0207	.0226	.0246	.0267
42	9.62	.00348	.00422	.00502	.00589	.00683	.00784	.00892	.01007	.01129	.0126	.0139	.0154	.0169	.0184	.0201	.0218
48	12.57	.00292	.00353	.00420	.00493	.00572	.00656	.00747	.00843	.00945	.01053	.01166	.0129	.0141	.0154	.0168	.0182
54	15.90	.00249	.00302	.00359	.00421	.00488	.00561	.00638	.00720	.00808	.00900	.00997	.01099	.0121	.0132	.0144	.0156
60	19.63	.00217	.00262	.00312	.00366	.00424	.00487	.00554	.00626	.00702	.00782	.00866	.00955	.01048	.0115	.0125	.0135

HEAD LOSS COEFFICIENT, K_c, FOR SQUARE CONDUIT FLOWING FULL $K_c = \frac{29.16\,n^2}{r^{\frac{4}{3}}}$

Conduit Size feet	Flow area sq. ft.	MANNING'S COEFFICIENT OF ROUGHNESS "n"				
		0.012	0.013	0.014	0.015	0.016
2×2	4.00	0.01058	0.01242	0.01440	0.01653	0.01880
$2\frac{1}{2}\times2\frac{1}{2}$	6.25	0.00786	0.00922	0.01070	0.01228	0.01397
3×3	9.00	.00616	.00723	.00839	.00963	.01096
$3\frac{1}{2}\times3\frac{1}{2}$	12.25	.00502	.00589	.00683	.00784	.00892
4×4	16.00	.00420	.00493	.00572	.00656	.00746
$4\frac{1}{2}\times4\frac{1}{2}$	20.25	.00359	.00421	.00488	.00561	.00638
5×5	25.00	.00312	.00366	.00425	.00487	.00554
$5\frac{1}{2}\times5\frac{1}{2}$	30.25	.00275	.00322	.00374	.00429	.00488
6×6	36.00	.00245	.00287	.00333	.00382	.00435
$6\frac{1}{2}\times6\frac{1}{2}$	42.25	.00220	.00258	.00299	.00343	.00391
7×7	49.00	.00199	.00234	.00271	.00311	.00354
$7\frac{1}{2}\times7\frac{1}{2}$	56.25	.00182	.00213	.00247	.00284	.00323
8×8	64.00	.00167	.00196	.00227	.00260	.00296
$8\frac{1}{2}\times8\frac{1}{2}$	72.25	.00154	.00180	.00209	.00240	.00273
9×9	81.00	.00142	.00167	.00194	.00223	.00253
$9\frac{1}{2}\times9\frac{1}{2}$	90.25	.00133	.00156	.00180	.00207	.00236
10×10	100.00	.00124	.00145	.00168	.00193	.00220

$$H_1 = (K_p \text{ or } K_c)\, L\, \frac{v^2}{2g}$$

Nomenclature:

- a = Cross-sectional area of flow in sq. ft.
- d_i = Inside diameter of pipe in inches.
- g = Acceleration of gravity = 32.2 ft. per sec.
- H_1 = Loss of head in feet due to friction in length L.
- K_c = Head loss coefficient for square conduit flowing full.
- K_p = Head loss coefficient for circular pipe flowing full.
- L = Length of conduit in feet.
- n = Manning's coefficient of roughness.
- Q = Discharge or capacity in cu. ft. per sec.
- r = Hydraulic radius in feet.
- v = Mean velocity in ft. per sec.

Example 1: Compute the head loss in 300 ft. of 24 in. diam. concrete pipe flowing full and discharging 30 c.f.s. Assume $n = 0.015$

$$v = \frac{Q}{a} = \frac{30}{3.14} = 9.55 \text{ f.p.s.}; \quad \frac{v^2}{2g} = \frac{(9.55)^2}{64.4} = 1.42 \text{ ft.}$$

$$H_1 = K_p L \frac{v^2}{2g} = 0.0165 \times 300 \times 1.42 = 7.03 \text{ ft.}$$

Example 2: Compute the discharge of a 250 ft., 3×3 square conduit flowing full if the loss of head is determined to be 2.25 ft. Assume $n = 0.014$.

$$H_1 = K_c L \frac{v^2}{2g}; \quad \frac{v^2}{2g} = \frac{H_1}{K_c L} = \frac{2.25}{0.00839 \times 250} = 1.073 \text{ ft.}$$

$$v = \sqrt{64.4 \times 1.073} = 8.31; \quad Q = 9 \times 8.31 = 74.8 \text{ c f s.}$$

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief

ENGINEERING STANDARDS UNIT

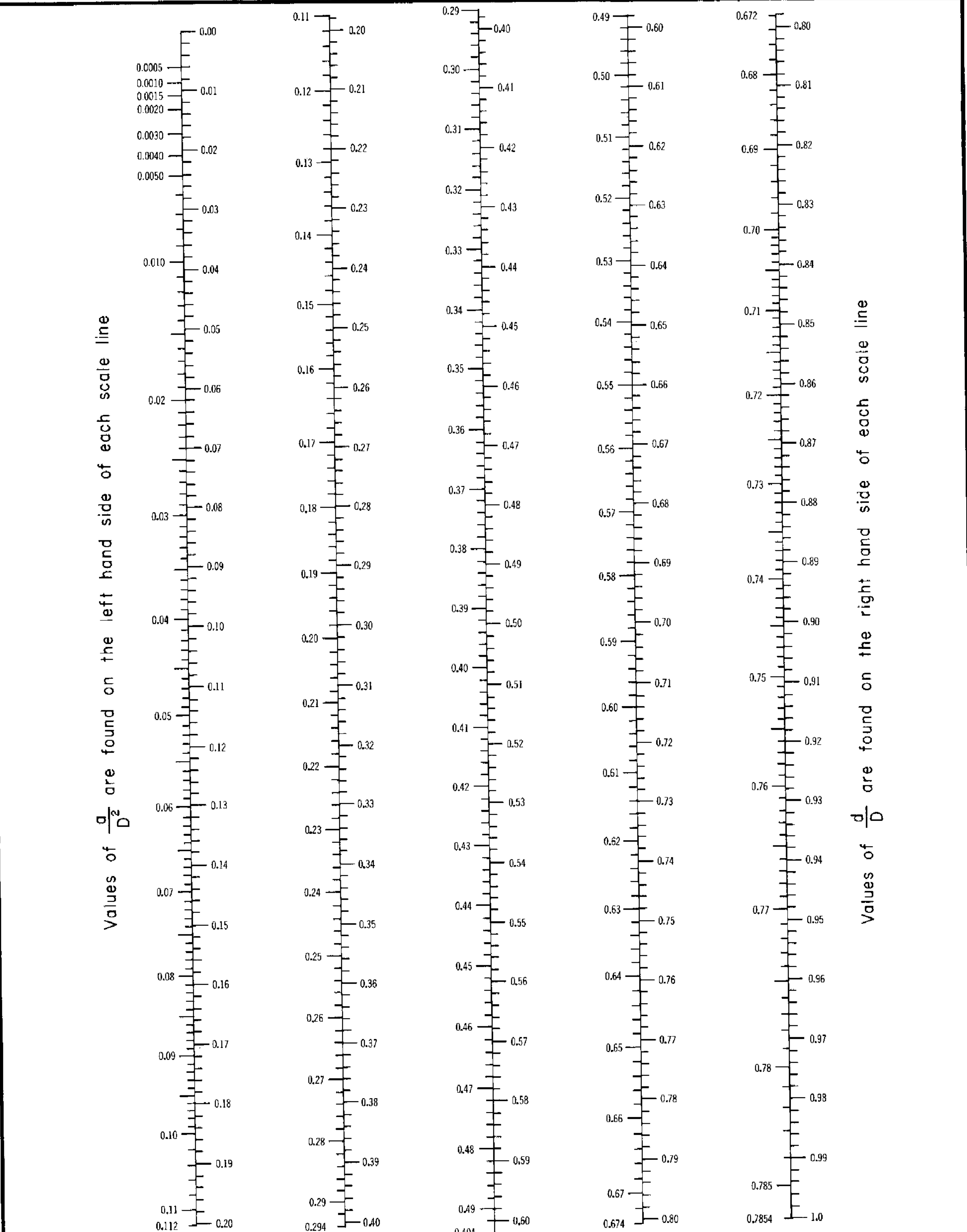
STANDARD DWG. NO. ES - 42

SHEET 1 OF 1

DATE 7-17-50

Revised 4-11-51

HYDRAULICS: FLOW AREAS ($a - ft^2$) IN CIRCULAR CONDUITS FOR VARIOUS DEPTHS OF FLOW (d - feet)



REFERENCE

This drawing was prepared by Richard M. Matthews of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE

SOIL CONSERVATION SERVICE

ENGINEERING DIVISION - DESIGN SECTION

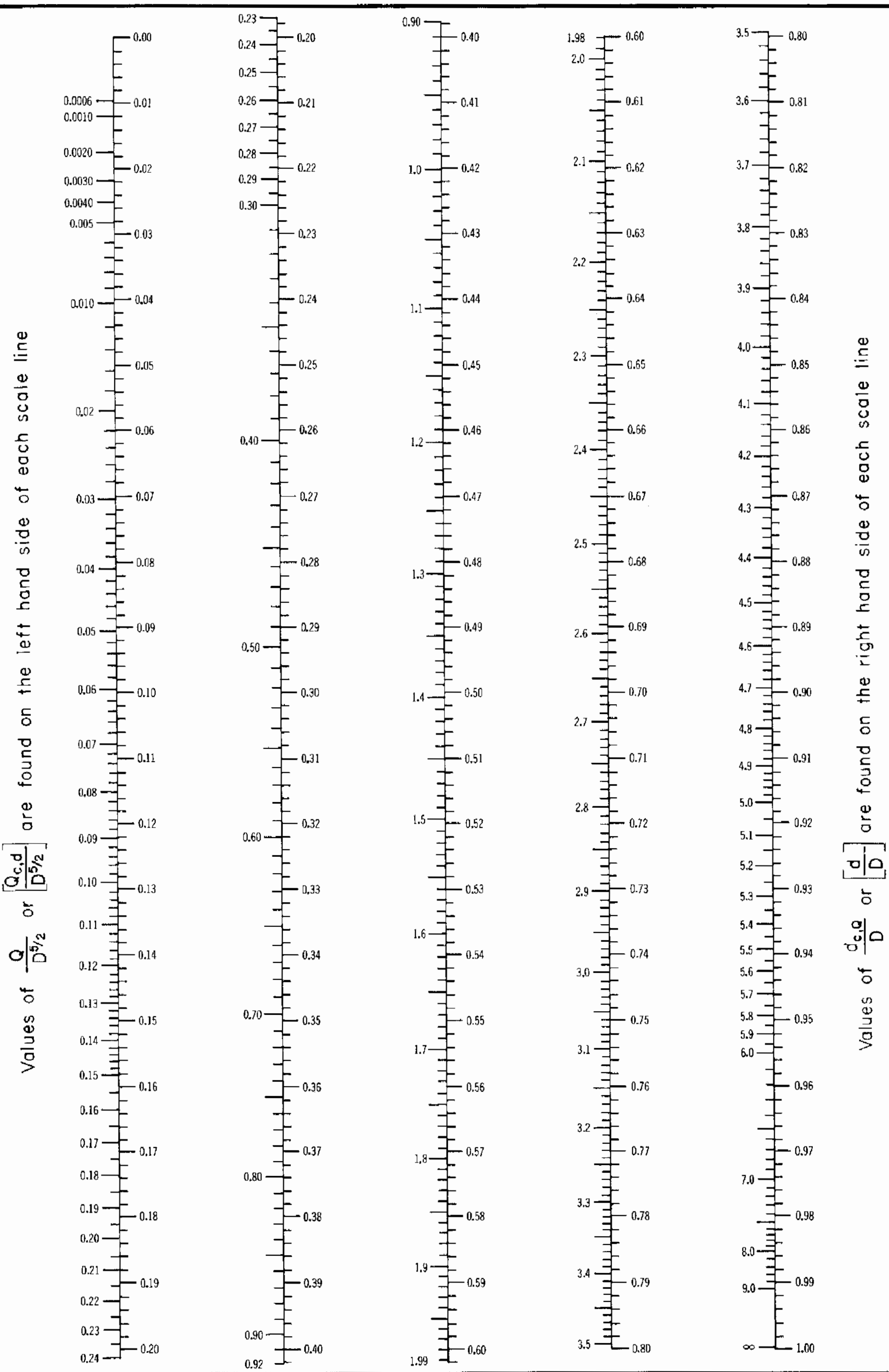
STANDARD DRAWING NO.

ES-97

SHEET 1 OF 7

DATE 1-6-55

HYDRAULICS: CIRCULAR CONDUITS; Critical discharges ($Q_{c,d}$ – cfs) corresponding to various depths (d – in ft) and critical depths ($d_{c,Q}$ – ft) corresponding to various discharges (Q – cfs)

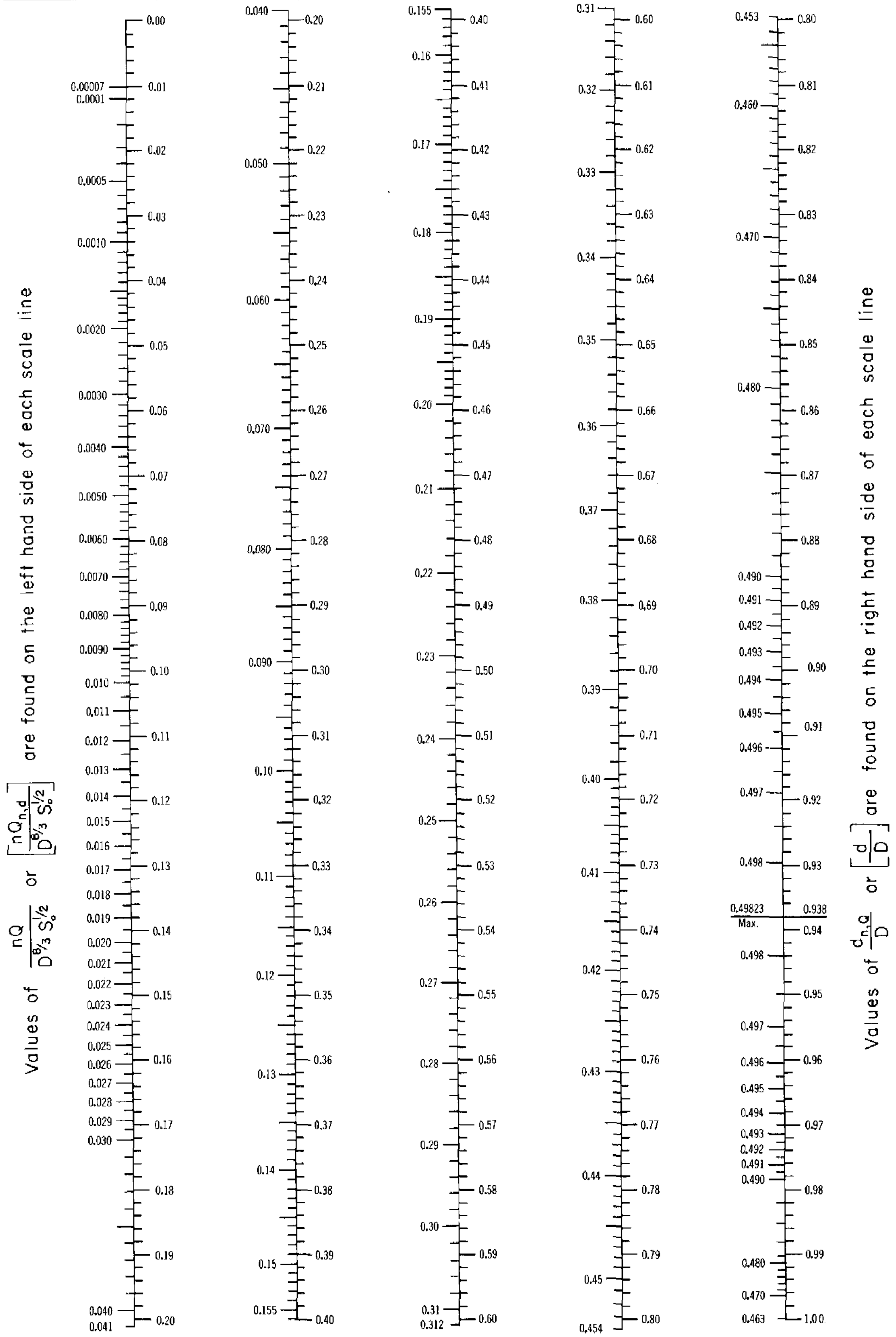


REFERENCE
This drawing was prepared by Richard M. Matthews of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DRAWING NO.
ES-97
SHEET 2 OF 7
DATE 1-6-55

HYDRAULICS: CIRCULAR CONDUITS; Normal discharges ($Q_{n,d}$ – cfs) corresponding to various depths (d – in ft) and normal depths ($d_{n,Q}$ – ft) corresponding to various discharges (Q – cfs)



REFERENCE
This drawing was prepared by Richard M. Matthews of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DRAWING NO. ES-97
SHEET 3 OF 7
DATE 1-6-55

HYDRAULICS: FLOW IN CIRCULAR CONDUITS; Definition of symbols and formulas.

DEFINITION OF SYMBOLS

a = Cross-sectional area of flow in sq ft
$a_{c,Q}$ = Critical area corresponding to the discharge Q in ft^2 = area corresponding to $d_{c,Q}$
$a_{n,Q}$ = Normal area corresponding to the discharge Q in ft^2 = area corresponding to $d_{n,Q}$
d = Depth of flow in ft
$d_{c,Q}$ = Critical depth corresponding to the discharge Q in ft
$d_{n,Q}$ = Normal depth corresponding to the discharge Q in ft
D = Inside diameter of circular conduits in ft
D_r = Required inside diameter with freeboard of circular conduit in ft
g = Acceleration due to gravity = 32.16 ft/sec^2
n = Manning's coefficient of roughness
Q = Discharge in cfs
$Q_{c,d}$ = Critical discharge in cfs
$Q_{n,d}$ = Normal discharge in cfs
r = Hydraulic radius in ft
s_c = Critical slope in ft/ft
s_o = Bottom slope of conduit in ft/ft
T = Top width of flow in ft
v = Velocity of flow in ft/sec
$v_{c,Q}$ = Critical velocity corresponding to the discharge Q in ft/sec = $\frac{Q}{a_{c,Q}}$
$v_{n,Q}$ = Normal velocity corresponding to the discharge Q in ft/sec = $\frac{Q}{a_{n,Q}}$

FORMULAS

$$\frac{a}{D^2} = \frac{1}{4}\left[\cos^{-1}\frac{D-2d}{D} - \left(\frac{D-2d}{D}\right)\sin\left(\cos^{-1}\frac{D-2d}{D}\right)\right]$$

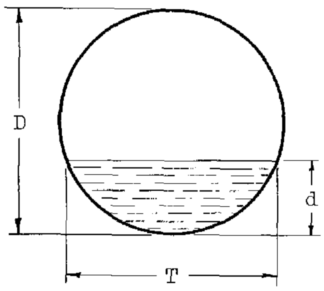
$$T = D\sin\left(\cos^{-1}\frac{D-2d}{D}\right)$$

$$\frac{Q_c^2}{g} = \frac{a_c^3}{T}$$

$$\frac{Q_c}{D^{5/2}} = \sqrt{\frac{g\left[\cos^{-1}\frac{D-2d_c}{D} - \left(\frac{D-2d_c}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]^3}{(4)^3\left[\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]}}$$

$$Q_n = \frac{1.486}{n} a_n r^{2/3} s_o^{1/2}$$

$$\frac{n\,Q_n}{D^{8/3}\,s_o^{1/2}} = \frac{1.486\left[\cos^{-1}\frac{D-2d_n}{D} - \left(\frac{D-2d_n}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_n}{D}\right)\right]^{5/3}}{(4)^{5/3}\left[\cos^{-1}\frac{D-2d_n}{D}\right]^{2/3}}$$



CIRCULAR SECTION

CONVERSIONS

$$\frac{\text{one cubic foot}}{\text{second}} = \frac{\text{448.8 gallons}}{\text{minute}}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DRAWING NO. ES-97 SHEET 4 OF 7 DATE 1-13-55

HYDRAULICS: FLOW IN CIRCULAR CONDUITS; Definition of symbols and formulas.

DEFINITION OF SYMBOLS

a = Cross-sectional area of flow in sq ft
$a_{c,Q}$ = Critical area corresponding to the discharge Q in ft^2 = area corresponding to $d_{c,Q}$
$a_{n,Q}$ = Normal area corresponding to the discharge Q in ft^2 = area corresponding to $d_{n,Q}$
d = Depth of flow in ft
$d_{c,Q}$ = Critical depth corresponding to the discharge Q in ft
$d_{n,Q}$ = Normal depth corresponding to the discharge Q in ft
D = Inside diameter of circular conduits in ft
D_r = Required inside diameter with freeboard of circular conduit in ft
g = Acceleration due to gravity = 32.16 ft/sec^2
n = Manning's coefficient of roughness
Q = Discharge in cfs
$Q_{c,d}$ = Critical discharge in cfs
$Q_{n,d}$ = Normal discharge in cfs
r = Hydraulic radius in ft
s_c = Critical slope in ft/ft
s_o = Bottom slope of conduit in ft/ft
T = Top width of flow in ft
v = Velocity of flow in ft/sec
$v_{c,Q}$ = Critical velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{c,Q}}$
$v_{n,Q}$ = Normal velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{n,Q}}$

FORMULAS

$$\frac{a}{D^2} = \frac{1}{4}\left[\cos^{-1}\frac{D-2d}{D} - \left(\frac{D-2d}{D}\right)\sin\left(\cos^{-1}\frac{D-2d}{D}\right)\right]$$

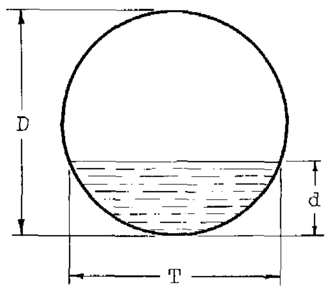
$$T = D\sin\left(\cos^{-1}\frac{D-2d}{D}\right)$$

$$\frac{Q_c^2}{g} = \frac{a_c^3}{T}$$

$$\frac{Q_c}{D^{5/2}} = \sqrt{\frac{g\left[\cos^{-1}\frac{D-2d_c}{D} - \left(\frac{D-2d_c}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]^3}{(4)^3\left[\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]}}$$

$$Q_n = \frac{1.486}{n}\,a_n r^{2/3}\,s_o^{1/2}$$

$$\frac{n\,Q_n}{D^{8/3}\,s_o^{1/2}} = \frac{1.486\left[\cos^{-1}\frac{D-2d_n}{D} - \left(\frac{D-2d_n}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_n}{D}\right)\right]^{5/3}}{(4)^{5/3}\left[\cos^{-1}\frac{D-2d_n}{D}\right]^{2/3}}$$



CIRCULAR SECTION

CONVERSIONS

$$\frac{\text{one cubic foot}}{\text{second}} = \frac{448.8\ \text{gallons}}{\text{minute}}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DRAWING NO. ES-97 SHEET 4 OF 7 DATE 1-13-55

HYDRAULICS: FLOW IN CIRCULAR CONDUITS; Examples

EXAMPLE 1

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The cross-sectional flow area a corresponding to a depth of flow d = 0.86 ft.

Solution: Solving for the flow area a when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $a \div D^2$ is 0.323. (From Sheet 1)

$$a = (0.323)(2.0)^2 = 1.292 \text{ ft}^2$$

EXAMPLE 2

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The depth of flow d corresponding to the cross-sectional area $a = 1.10 \text{ ft}^2$.

Solution: Solving for the depth of flow d when $a \div D^2 = 1.10 \div (2.0)^2 = 0.275$, the corresponding value for $d \div D$ is 0.381. (From Sheet 1)

$$d = (0.381)(2.0) = 0.762 \text{ ft}$$

EXAMPLE 3

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The critical discharge $Q_{c,d}$ corresponding to the depth of flow d = 0.86 ft.

Solution: Solving for the critical discharge $Q_{c,d}$ when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $Q_{c,d} \div D^{5/2}$ is 1.046. (From Sheet 2)

$$Q_{c,d} = (1.046)(2.0)^{5/2} = 5.917 \text{ cfs}$$

EXAMPLE 4

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The critical depth of flow $d_{c,Q}$ corresponding to the discharge Q = 5.60 cfs.

Solution: Solving for the critical depth $d_{c,Q}$ when $Q \div D^{5/2} = 5.60 \div (2.0)^{5/2} = 0.9915$, the corresponding value for $d_{c,Q} \div D$ is 0.418. (From Sheet 2)

$$d_{c,Q} = (0.418)(2.0) = 0.836 \text{ ft}$$

EXAMPLE 5

Given: A concrete circular conduit having a diameter D = 2.0 ft, n = 0.015, and $s_o = 0.0025$.

Determine: The normal discharge $Q_{n,d}$ corresponding to a depth of flow d = 0.86 ft.

Solution: Solving for the normal discharge $Q_{n,d}$ when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $nQ_{n,d} \div D^{8/3}s_o^{1/2}$ is 0.178. (From Sheet 3)

$$Q_{n,d} = \frac{(0.178)(2.0)^{8/3}(0.0025)^{1/2}}{0.015} = 3.768 \text{ cfs}$$

EXAMPLE 6

Given: A concrete circular conduit having a diameter D = 2.0 ft, n = 0.015, and $s_o = 0.0025$.

Determine: The normal depth of flow $d_{n,Q}$ corresponding to the discharge Q = 3.60 cfs.

Solution: Solving for the normal depth $d_{n,Q}$ when $nQ \div D^{8/3}s_o^{1/2} = (0.015)(3.60) \div (2.0)^{8/3}(0.0025)^{1/2} = 0.1701$, the corresponding value for $d_{n,Q} \div D$ is 0.419. (From Sheet 3)

$$d_{n,Q} = (0.419)(2.0) = 0.838 \text{ ft}$$

EXAMPLE 7

Given: A semi-circular metal flume having a diameter of D = 1.91 ft. Manning's roughness coefficient n = 0.012.

Determine: The critical slope of this flume for a discharge of Q = 1700 gpm = 3.788 cfs.

Solution: The critical slope s_c corresponding to a discharge Q is that bottom slope $s_o = s_c$ which will cause the critical depth $d_{c,Q}$ and the normal depth $d_{n,Q}$ to be equal. Solving for critical depth $d_{c,Q}$ corresponding to Q = 3.788 cfs.

When $\frac{Q}{D^{5/2}} = \frac{3.788}{(1.91)^{5/2}} = 0.7513$, the corresponding value for $\frac{d_{c,Q}}{D}$ is 0.3620. (From Sheet 2)

When $\frac{d_{n,Q}}{D} = 0.3620$, the corresponding value for $\frac{nQ}{D^{8/3}s_c^{1/2}}$ is 0.1298, (From Sheet 3) or

$$s_c = \left[\frac{nQ}{0.1298D^{8/3}}\right]^2 = \left[\frac{(0.012)(3.788)}{(0.1298)(1.91)^{8/3}}\right]^2 = 0.003883 \text{ ft/ft}$$

EXAMPLE 8

Given: The problem of designing a straight, semi-circular metal flume (n = 0.012). The flume is to convey a maximum discharge of Q = 1700 gpm = 3.788 cfs with a *minimum freeboard* of 6 percent of the diameter D of the flume and a bottom slope s_o which will permit a maximum velocity v equal to 80 percent of the critical velocity $v_{c,Q}$ corresponding to the discharge Q is to be provided. The actual velocity v is not to exceed 4 ft/sec. The flume is to be constructed so that the depth of flow at the outlet end of the flume will be equal to the calculated normal depth $d_{n,Q}$ corresponding to the discharge Q if $v_{n,Q} \leq 4$ ft/sec. This insures that normal flow conditions will exist in the flume (see ES-38, Case A).

Determine: a(1) The required diameter D_r of the flume which will satisfy, simultaneously, the stated freeboard and velocity criteria.

(2) The diameter D of a standard size flume to be used.

b(1) The critical velocity $v_{c,Q}$ corresponding to the discharge Q in the flume having the diameter D.

(2) The maximum bottom slope s_o of the flume having a diameter D determined in a(2).

(3) The normal depth of flow $d_{n,Q}$ corresponding to the discharge Q.

(4) The percent freeboard of diameter D.

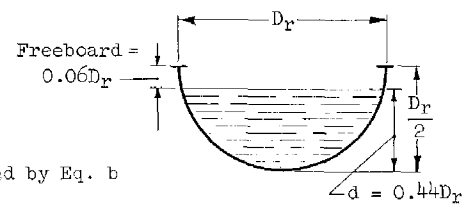
c(1) The *minimum* slope s_o of the flume having a diameter D determined in a(2).

(2) The actual velocity existing in the flume with the minimum slope.

(3) The ratio $v/v_{c,Q}$.

Solution: a(1) Solving for the required diameter D_r of the flume which satisfies the stated freeboard and velocity criteria. The depth of flow is 6 percent of D_r less than $\frac{D_r}{2}$ or $d = \frac{D_r}{2} - 0.06D_r$ or $\frac{d}{D_r} = 0.44$. The critical velocity $v_{c,Q}$ corresponding to the discharge Q is

$$v_{c,Q} = \frac{Q}{a_{c,Q}} \quad \text{(Eq. a)}$$



The actual velocity v is

$$v = \frac{Q}{a} \quad \text{(Eq. b)}$$

When $\frac{d}{D_r} = 0.44$, the corresponding value of $\frac{a}{D_r^2}$ is 0.333 and by Eq. b

$$v = \frac{Q}{(0.333)D_r^2} \quad \text{(Eq. c)}$$

On substituting for v and $v_{c,Q}$ (see Eqs. a and c) and the stated criterion $v = 0.80v_{c,Q}$, obtain (It will be assumed that v is less than 4 ft/sec)

$$\frac{Q}{(0.333)D_r^2} = 0.80\frac{Q}{a_{c,Q}} \quad \text{or} \quad \frac{a_{c,Q}}{D_r^2} = 0.80(0.333) = 0.2664$$

When $\frac{a_{c,Q}}{D_r^2} = 0.2664$, the corresponding value of $\frac{d_{c,Q}}{D_r}$ is 0.3727. When $\frac{d_{c,Q}}{D_r} = 0.3727$, the corresponding value of $\frac{Q}{D_r^{5/2}}$ is 0.795.

$$D_r^{5/2} = \frac{Q}{0.795} = \frac{3.788}{0.795} = 4.7648$$

$$D_r = 1.867 \text{ ft}$$

Concluded on Sheet 7

Continuation from Sheet 6

There exists only one value of D_r which will satisfy both the criteria (1) $d = 0.44D_r$ and (2) $v = 0.80v_{c,Q}$ simultaneously, for a given value of Q, n, and cross-sectional shape.

(2) The value of D_r is not a standard diameter size. The next standard diameter D size greater than D_r is to be chosen. Use 23-inch sheeting or D = 1.91 ft. By changing the diameter D to a greater number than D_r, both criteria cannot be satisfied simultaneously. Two extremes in the bottom slopes can be obtained by:

(1) fixing the maximum velocity in accordance to the stated criteria and allowing the freeboard to increase above the criterion.

(2) fixing the minimum freeboard to 6 percent of D and allowing the velocity to be decreased from that given by the criterion.

b(1) Solving for the critical velocity $v_{c,Q}$ corresponding to the discharge Q. When $\frac{Q}{D^{5/2}} = \frac{3.788}{(1.91)^{5/2}} = 0.75125$, the corresponding value for $\frac{d_{c,Q}}{D}$ is 0.3620. (From Sheet 2) When $\frac{d_{c,Q}}{D} = 0.3620$, the corresponding value for $\frac{a_{c,Q}}{D^2}$ is 0.2565.

$$a_{c,Q} = 0.2565D^2 = 0.2565\ (1.91)^2 = 0.9357\ \text{ft}^2$$

$$v_{c,Q} = \frac{Q}{a_{c,Q}} = \frac{3.788}{0.9357} = 4.048\ \text{ft/sec}$$

(2) The maximum bottom slope is determined by fixing the maximum permissible velocity.

$$v_{n,Q} = 0.80v_{c,Q} = 0.80\ (4.048) = 3.238\ \text{ft/sec}$$

$$a_{n,Q} = \frac{Q}{v_{n,Q}} = \frac{3.788}{3.238} = 1.170\ \text{ft}^2$$

When $\frac{a_{n,Q}}{D^2} = \frac{1.170}{(1.91)^2} = 0.3207$, the corresponding value for $\frac{d_{n,Q}}{D}$ is 0.4277. (From Sheet 1) When $\frac{d_{n,Q}}{D} = 0.4277$, the corresponding value for $\frac{nQ}{D^{8/3}\ s_o^{1/2}}$ is 0.1763, (From Sheet 3) or

$$s_o = \left[\frac{nQ}{D^{8/3}(0.1763)}\right]^2 = \left[\frac{(0.012)(3.788)}{(1.91)^{8/3}(0.1763)}\right]^2 = 0.002105$$

(3) Solving for the normal depth of flow $d_{n,Q}$ corresponding to the discharge Q is (see above)

$$d_{n,Q} = 0.4277D = 0.4277\ (1.91) = 0.817\ \text{ft}$$

(4) The percent freeboard is

$$\frac{\left[\frac{D}{2} - d_{n,Q}\right]100}{D} = \frac{\left[\frac{1.91}{2} - 0.817\right]100}{1.91} = 7.23\%$$

c(1) When the minimum slope is desired the minimum velocity is required, i.e., the maximum flow area. The maximum flow area permissible by the stated criteria corresponds to a depth of flow equal to 0.44D. Solving for the minimum slope s_o of the flume having a depth of flow d = 0.44D. When $\frac{d}{D} = 0.44$, the corresponding value for $\frac{nQ_{n,d}}{D^{8/3}\ s_o^{1/2}}$ is 0.1853

$$s_o = \left[\frac{nQ}{D^{8/3}(0.1853)}\right]^2 = \left[\frac{(0.012)(3.788)}{(1.91)^{8/3}(0.1853)}\right]^2 = 0.001905$$

(2) Solving for the actual velocity at flow depth d = 0.44D. When $\frac{d}{D} = 0.44$, the corresponding value for $\frac{a}{D^2} = 0.333$.

$$a = 0.333D^2 = 0.333(1.91)^2 = 1.215\ \text{ft}^2$$

$$v = \frac{Q}{a} = \frac{3.788}{1.215} = 3.118\ \text{ft/sec}$$

(3 Solving for the ratio $\frac{v}{v_{c,Q}}$

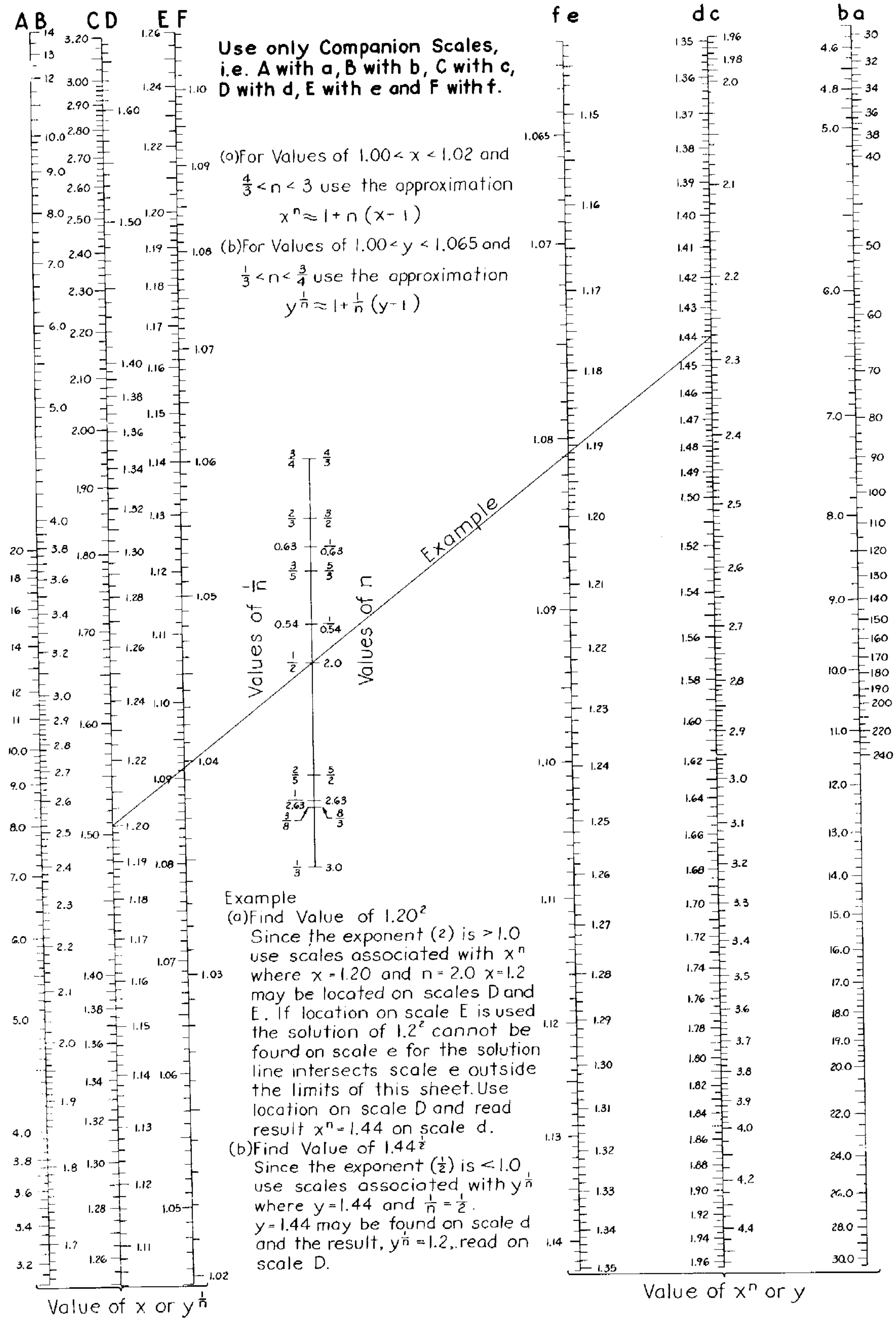
$$\frac{v}{v_{c,Q}} = \frac{3.118}{4.048} = 77.05\%$$

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND y > 1

Use only Companion Scales, i.e. A with a, B with b, C with c, D with d, E with e and F with f.

(a) For Values of $1.00 < x < 1.02$ and $\frac{4}{3} < n < 3$ use the approximation
$$x^n \approx 1 + n(x - 1)$$

(b) For Values of $1.00 < y < 1.065$ and $\frac{1}{3} < n < \frac{3}{4}$ use the approximation
$$y^{\frac{1}{n}} \approx 1 + \frac{1}{n}(y - 1)$$



Example

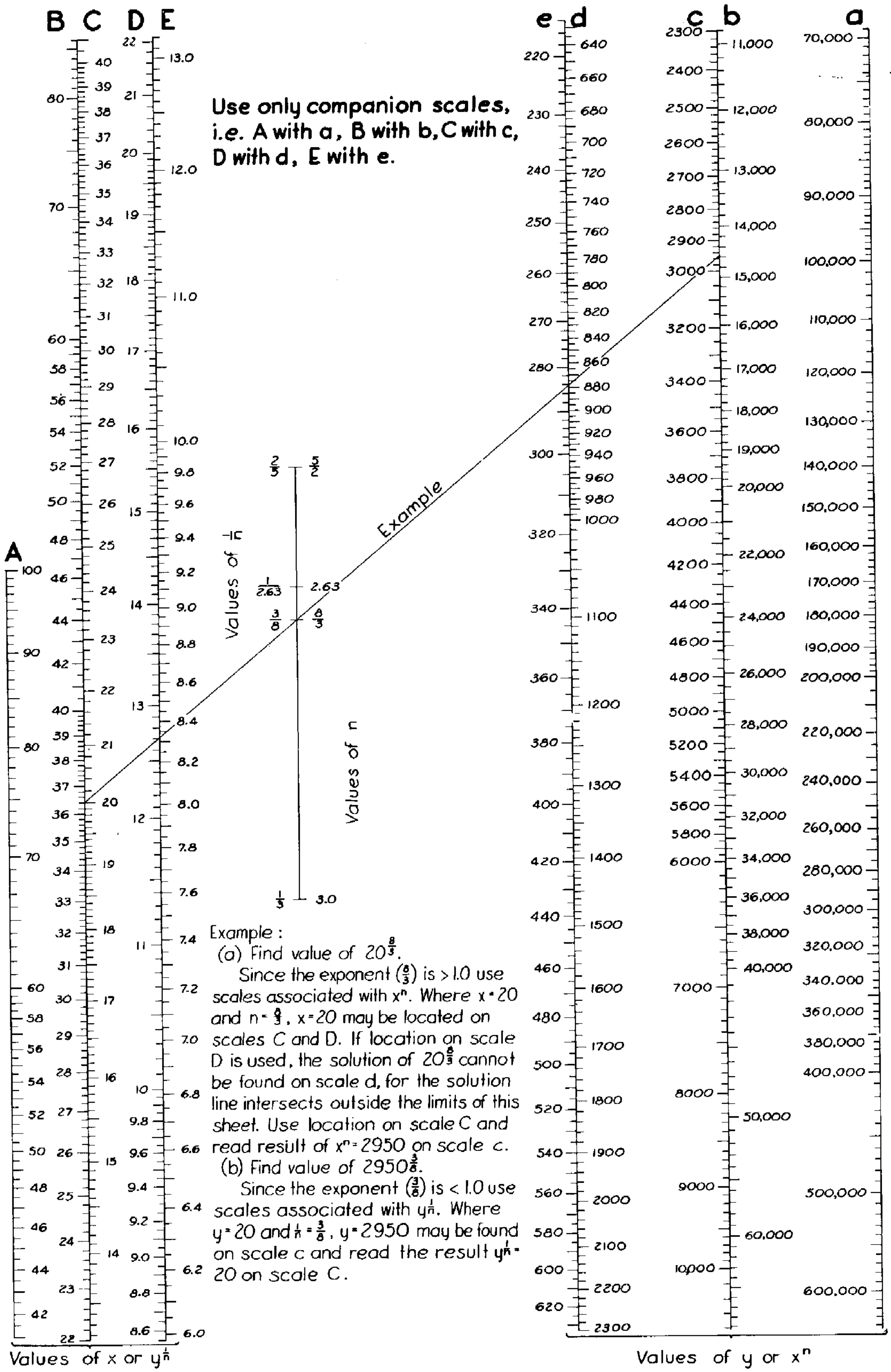
(a) Find Value of 1.20^2
Since the exponent (2) is > 1.0 use scales associated with x^n where $x = 1.20$ and $n = 2.0$ $x = 1.2$ may be located on scales D and E. If location on scale E is used the solution of 1.2^2 cannot be found on scale e for the solution line intersects scale e outside the limits of this sheet. Use location on scale D and read result $x^n = 1.44$ on scale d.

(b) Find Value of $1.44^{\frac{1}{2}}$
Since the exponent ($\frac{1}{2}$) is < 1.0 use scales associated with $y^{\frac{1}{n}}$ where $y = 1.44$ and $\frac{1}{n} = \frac{1}{2}$.
$y = 1.44$ may be found on scale d and the result, $y^{\frac{1}{n}} = 1.2$, read on scale D.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 37 SHEET 1 OF 3 DATE 6-3-50

Revised 6-4-51

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND y > 1



Use only companion scales, i.e. A with a, B with b, C with c, D with d, E with e.

Example :

(a) Find value of $20^{\frac{8}{3}}$.

Since the exponent ($\frac{8}{3}$) is > 1.0 use scales associated with x^n. Where x = 20 and n = $\frac{8}{3}$, x = 20 may be located on scales C and D. If location on scale D is used, the solution of $20^{\frac{8}{3}}$ cannot be found on scale d, for the solution line intersects outside the limits of this sheet. Use location on scale C and read result of x^n = 2950 on scale c.

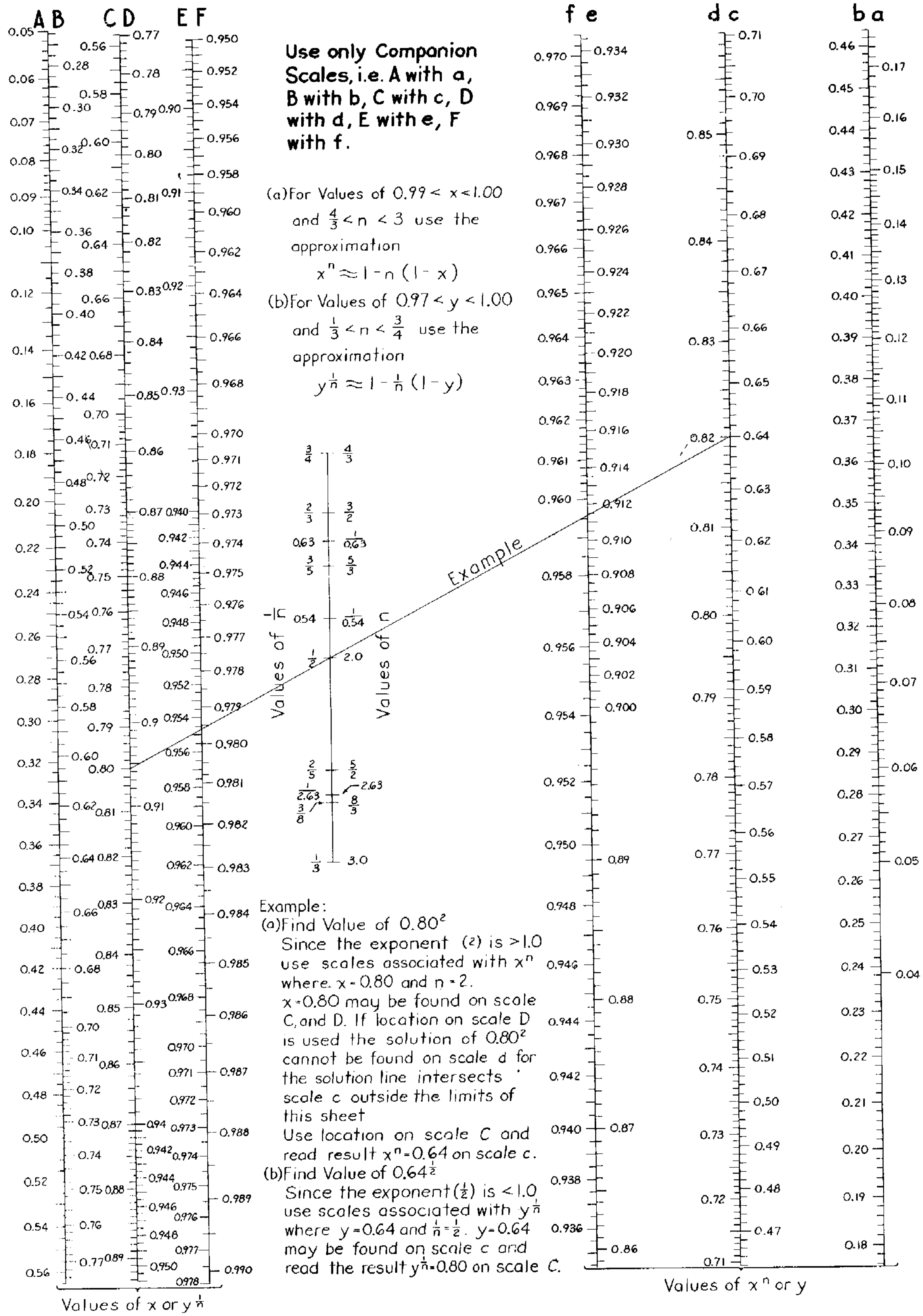
(b) Find value of $2950^{\frac{3}{8}}$.

Since the exponent ($\frac{3}{8}$) is < 1.0 use scales associated with $y^{\frac{1}{n}}$. Where y = 20 and $\frac{1}{n} = \frac{3}{8}$, y = 2950 may be found on scale c and read the result $y^{\frac{1}{n}}$ = 20 on scale C.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-37 SHEET 2 OF 3 DATE 6-3-50

Revised 6-4-51

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND $y<1$



Use only Companion Scales, i.e. A with a, B with b, C with c, D with d, E with e, F with f.

(a) For Values of $0.99 < x < 1.00$ and $\frac{4}{3} < n < 3$ use the approximation

$$x^n \approx 1 - n(1-x)$$

(b) For Values of $0.97 < y < 1.00$ and $\frac{1}{3} < n < \frac{3}{4}$ use the approximation

$$y^{\frac{1}{n}} \approx 1 - \frac{1}{n}(1-y)$$

Example:

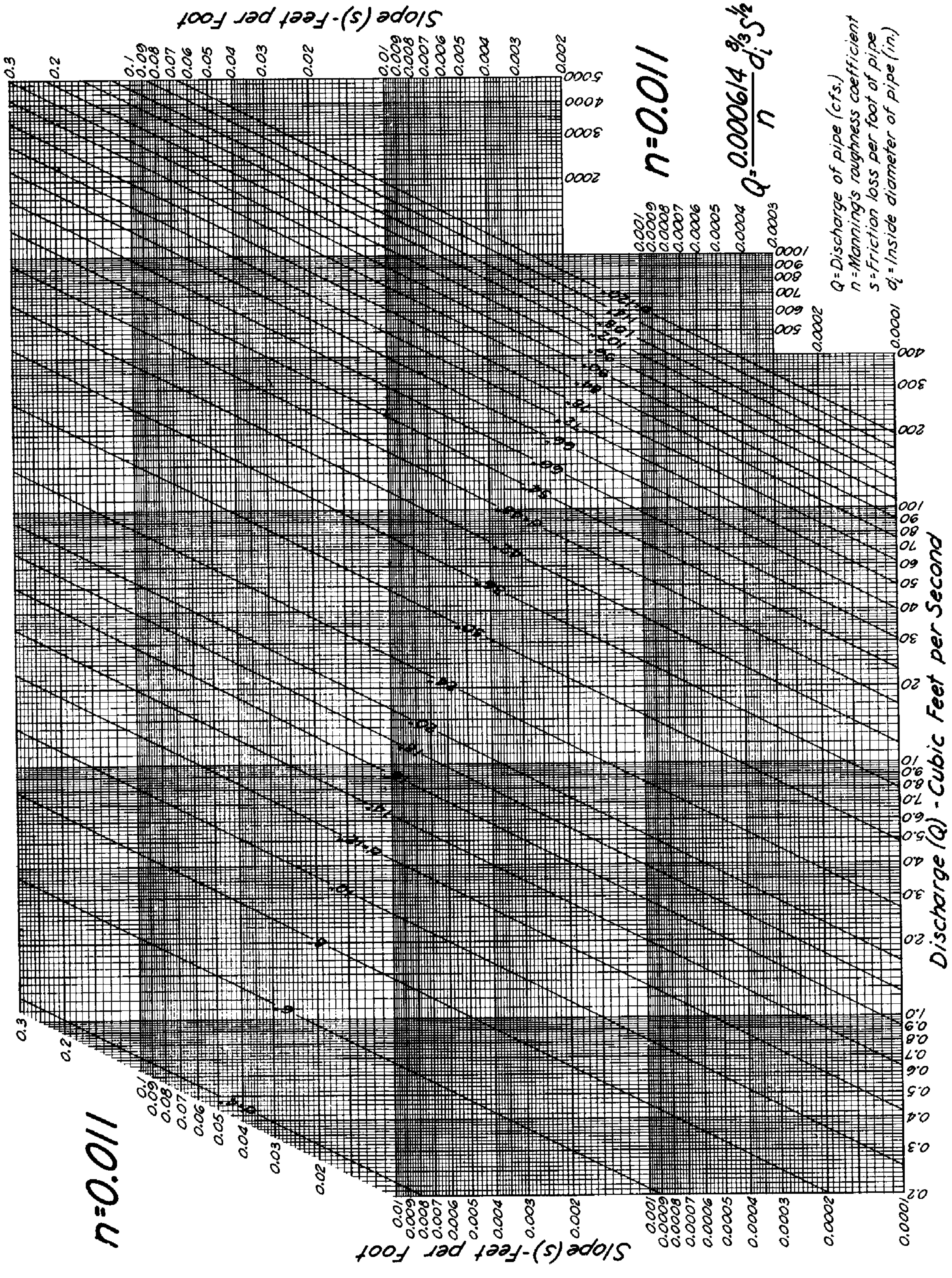
(a) Find Value of 0.80^2
Since the exponent (2) is >1.0 use scales associated with x^n where $x = 0.80$ and $n = 2$.
$x = 0.80$ may be found on scale C, and D. If location on scale D is used the solution of 0.80^2 cannot be found on scale d for the solution line intersects scale c outside the limits of this sheet.
Use location on scale C and read result $x^n = 0.64$ on scale c.

(b) Find Value of $0.64^{\frac{1}{2}}$
Since the exponent ($\frac{1}{2}$) is <1.0 use scales associated with $y^{\frac{1}{n}}$ where $y = 0.64$ and $\frac{1}{n} = \frac{1}{2}$. $y = 0.64$ may be found on scale c and read the result $y^{\frac{1}{n}} = 0.80$ on scale C.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 37 SHEET 3 OF 3 DATE 6 - 3 - 50

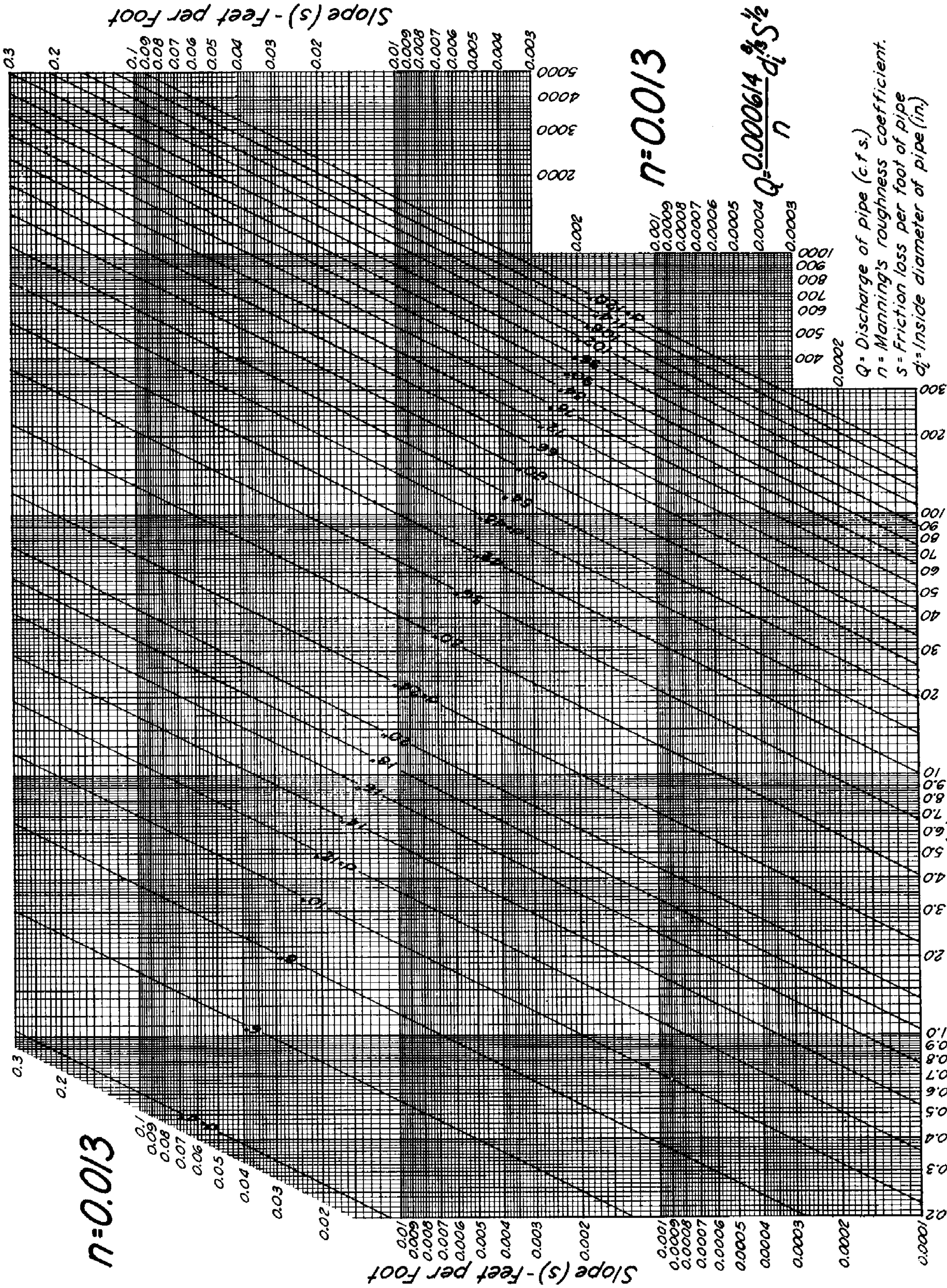
Revised 6-4-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



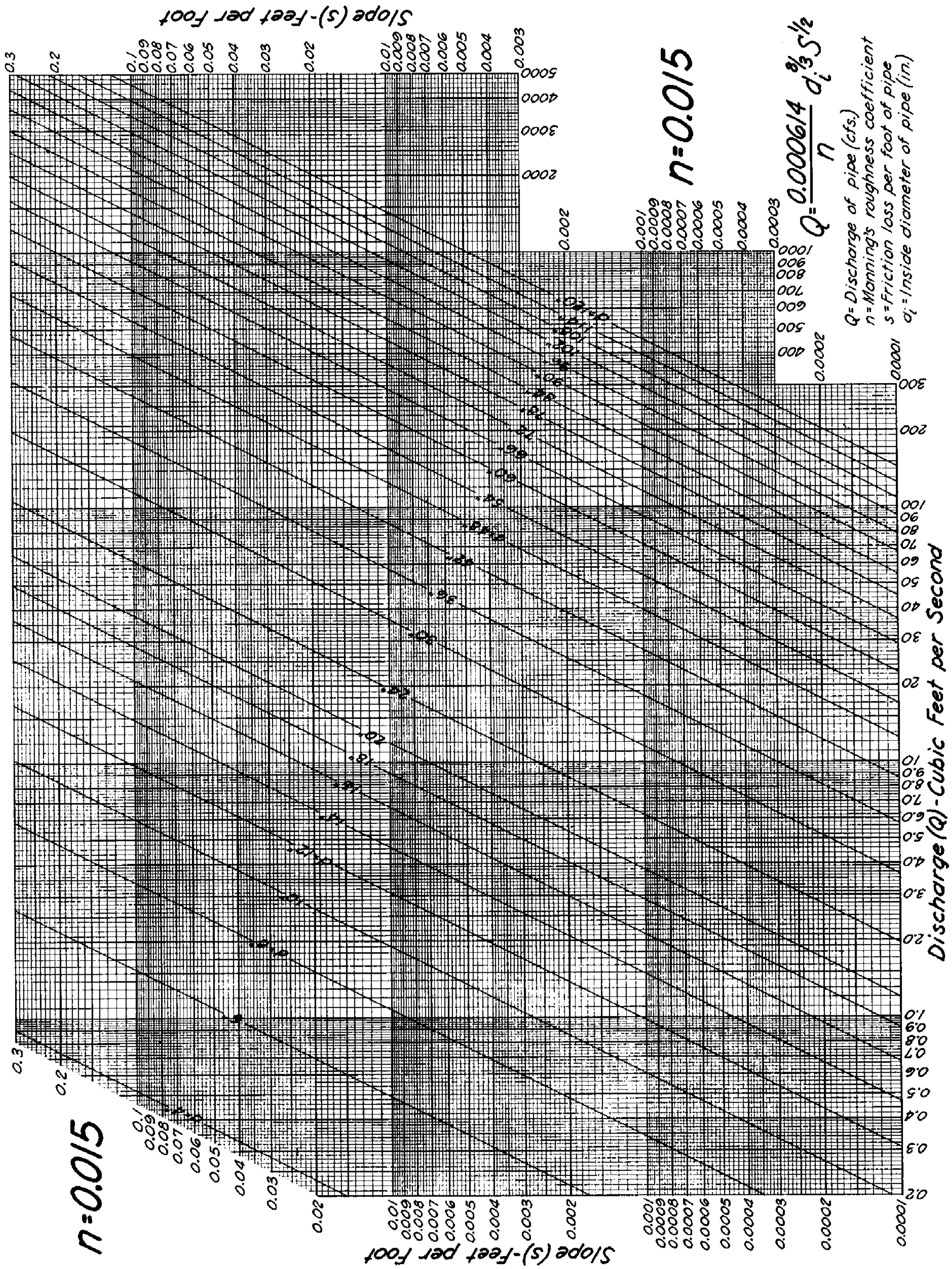
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 1 OF 4 DATE 4-27-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



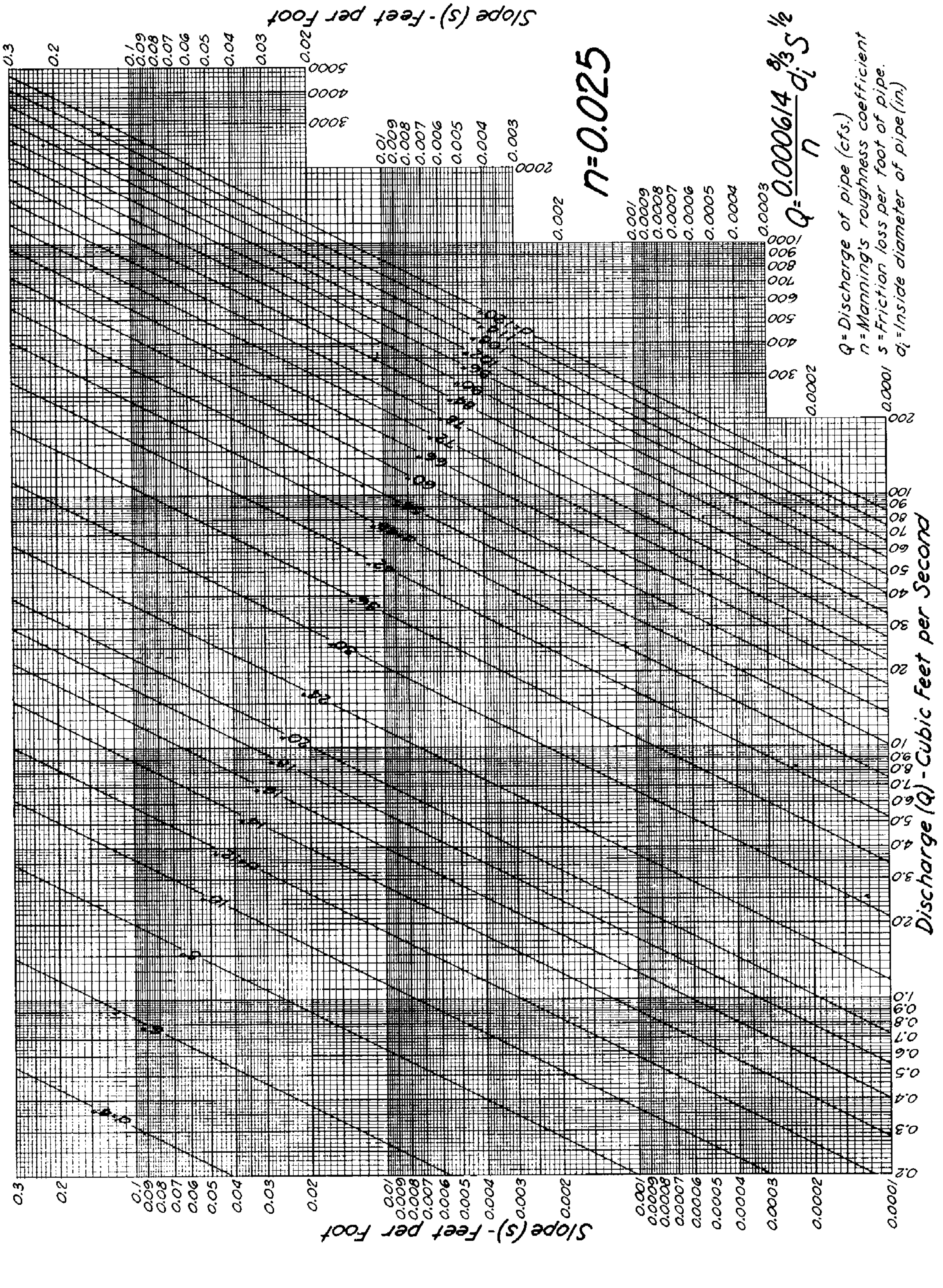
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 2 OF 4 DATE 4-27-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 3 OF 4 DATE 4-27-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 4 OF 4 DATE 4-27-51

Since Q = av, equation (5.5-14) may be converted to the following formula for discharge in any conduit:

$$Q = 1.318\ a\ C\ r^{0.63}\ s^{0.54} \qquad (5.5\text{-}15)$$

Substitution of a and r in terms of inside diameter of pipe in inches in equation (5.5-15) gives the following general formula for discharge in circular pipes:

$$Q/C = 0.0006273\ d_i^{2.63}\ s^{0.54} \qquad (5.5\text{-}16)$$

Graphical solutions of equation (5.5-16) for standard pipe ranging from 1 to 12 inches in diameter and a wide range in slope may be made by drawing ES-40. The 0.63 and 0.54 powers of numbers for use in the above forms of Hazen-Williams formula may be computed by drawing ES-37.

Values of C for different types of pipe are given in table 5.5-2.

TABLE 5.5-2. VALUES OF HAZEN-WILLIAMS C

Description of Pipe	C
1. Very smooth pipe; straight alignment - - - - - - - - - - - - -	140
2. Very smooth pipe; slight curvature - - - - - - - - - - - - - -	130
3. Cast iron, uncoated - new - - - - - - - - - - - - - - - - - -	130
5 years old - - - - - - - - - - - - - -	120
10 years old - - - - - - - - - - - - - -	110
15 years old - - - - - - - - - - - - - -	100
20 years old - - - - - - - - - - - - - -	90
30 years old - - - - - - - - - - - - - -	80
coated - all ages - - - - - - - - - - - - - - - -	130
4. Steel pipe, welded, new -	130
(Same deterioration with age as cast iron, uncoated)	
For permanent installation use - - - - - - - - - - - - - - - - -	100
5. Wrought iron or standard galvanized steel - diam. 12 in. up	110
4 to 12 in.	100
4 in. down	80
6. Brass or lead, new -	140
7. Concrete, very smooth, excellent joints - - - - - - - - - - -	140
smooth, good joints - - - - - - - - - - - - - -	120
rough -	110
8. Vitrified -	110
9. Smooth wooden or wood stave - - - - - - - - - - - - - - - - -	120
10. Asbestos, cement -	140
11. Corrugated pipe -	60

Note: Pipes of small diameter, old age, and very rough inside surface, may give values as low as C = 40

5.4 Other Losses. In addition to the friction head losses there are other losses of energy which occur as the result of turbulence created by changes in velocity and direction of flow. To facilitate their inclusion in Bernoulli's energy equation, such losses are commonly expressed in terms of the mean velocity head at some specific cross section of the pipe.

These losses are sometimes called minor losses which may be a serious misnomer. In long pipe lines, the entrance loss, bend losses, etc., may be a relatively insignificant part of the total loss and in such cases can be ignored without introducing significant error. Such is not the case in many structures such as culverts, drop inlets, and siphons which are relatively short. Safe design practice requires an estimate of such losses. In case the estimate indicates that minor losses amount to 5 percent or more of the total head loss, they should be carefully evaluated and included in the flow calculations.

As velocities increase, careful determination of such minor losses becomes increasingly important; with a mean velocity of 30 feet per second, the neglect of an entrance loss of 0.5 $v^2/2g$ results in an error in head loss of 7 feet, whereas if the mean velocity is 3 feet per second, neglect of such an entrance loss results in an error of only 0.07 feet.

Data on minor losses most commonly required are contained in the following subsections.

5.4.1 Entrance Loss. Loss of head at the entrance of a pipe results from turbulence caused by the contraction of the flow cross section. It is expressed by the following equation:

$$H_o = K_o \frac{v^2}{2g} \qquad (5.5\text{-}17)$$

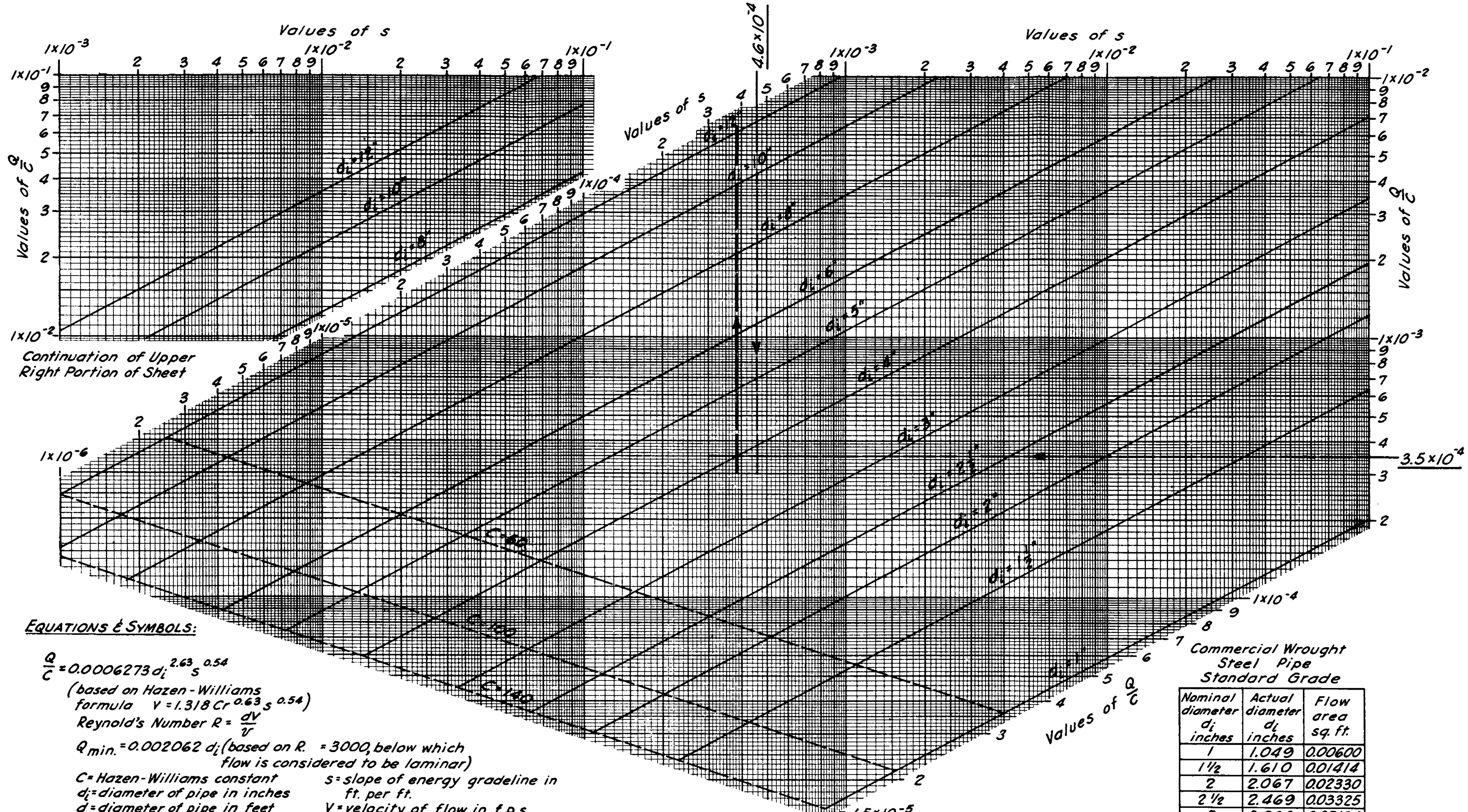
in which

H_o = head loss at the entrance in ft.
K_o = a coefficient dependent on the type of inlet.
v = velocity in pipe in fps.
g = acceleration of gravity in ft. per sec.^{2}

Values of K_o are given in the following table:

Type of Inlet	Value of K_o
Inward projecting	0.78
Sharp cornered	0.50
Bell mouth	0.04

HYDRAULICS: SOLUTION OF HAZEN - WILLIAMS FORMULA FOR ROUND PIPES



EQUATIONS & SYMBOLS:

$\frac{Q}{C} = 0.0006273\, d_i^{2.63}\, s^{0.54}$

(based on Hazen - Williams formula $V = 1.318\, C r^{0.63} s^{0.54}$)

Reynold's Number $R = \frac{dV}{\nu}$

$Q_{min.} = 0.002062\, d_i$ (based on R. = 3000, below which flow is considered to be laminar)

C = Hazen-Williams constant
d_i = diameter of pipe in inches
d = diameter of pipe in feet
Q = discharge in c.f.s.
r = hydraulic radius in feet
s = slope of energy gradeline in ft. per ft.
V = velocity of flow in f.p.s.
ν = kinematic viscosity in ft.^{2} per sec. (assumed to be 1.05×10^{-5} ft.^{2} per sec. for water at 70°F.)

Note: Dashed lines pass through minimum values of Q/C for the given C. For lesser values of Q/C, according to the assumptions of ν and R, there can be no assurance of turbulent flow.

Example: Find size pipe required to carry Q = 0.042 cfs with a maximum energy gradient S = 0.00046 if C = 120. Then $Q/C = \frac{0.042}{120} = 3.5 \times 10^{-4}$. Enter chart, with above values, and find d_i = 4 in.. Actual head loss in 4 in. pipe = 3.85×10^{-4} ft. per ft.

Commercial Wrought Steel Pipe Standard Grade

Nominal diameter d_i inches	Actual diameter d_i inches	Flow area sq. ft.
1	1.049	0.00600
1½	1.610	0.01414
2	2.067	0.02330
2½	2.469	0.03325
3	3.068	0.05134
4	4.026	0.08840
5	5.047	0.1389
6	6.065	0.2006
8	7.981	0.3474
10	10.020	0.5475
12	12.000	0.7854

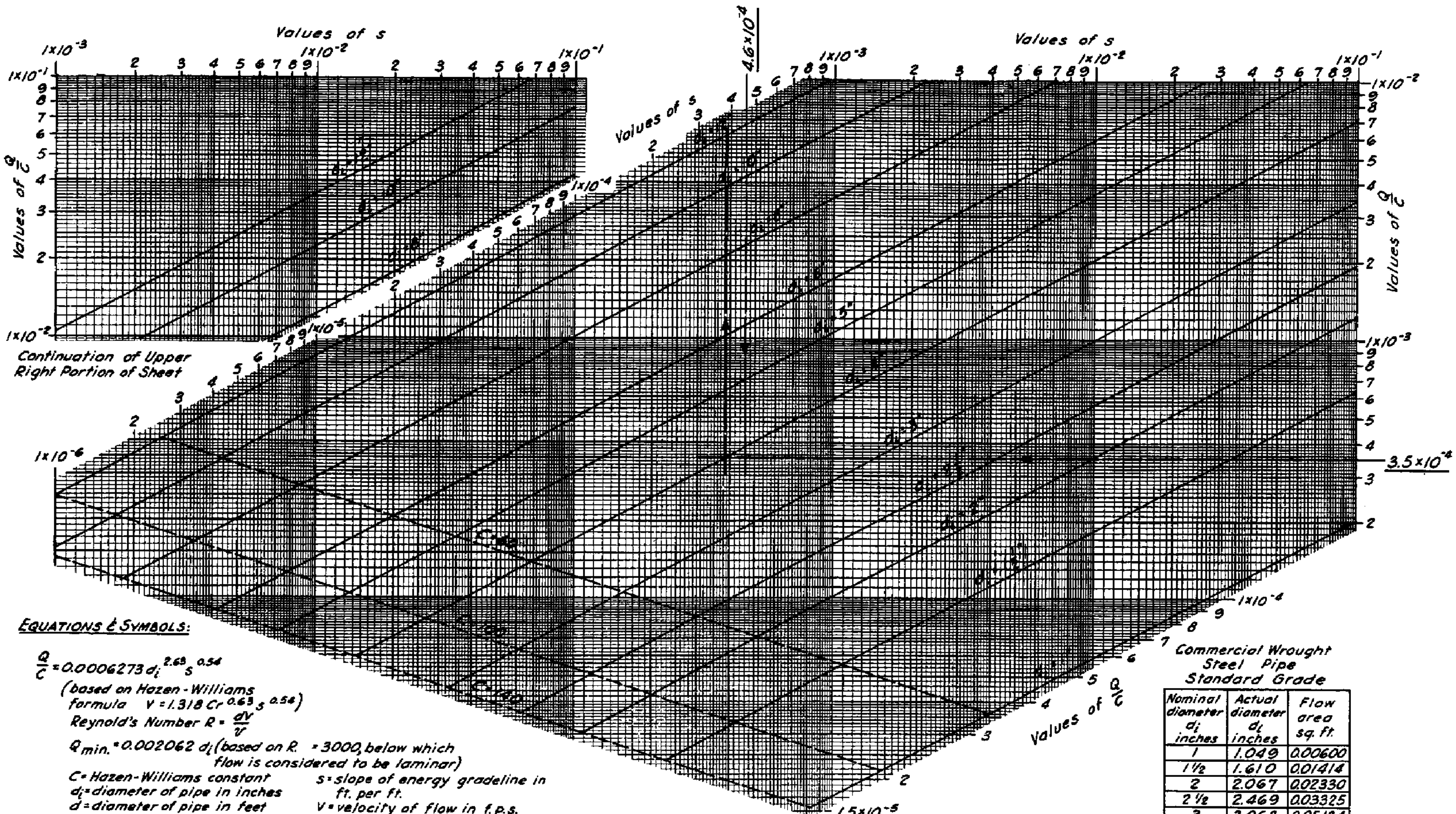
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-40
SHEET 1 OF 1
DATE 7-21-50

REVISED 3-7-51

HYDRAULICS: SOLUTION OF HAZEN-WILLIAMS FORMULA FOR ROUND PIPES



EQUATIONS & SYMBOLS:

$\frac{Q}{C} = 0.0006273\, d_i^{2.63} s^{0.54}$

(based on Hazen-Williams formula $V = 1.318\, C r^{0.63} s^{0.54}$)

Reynold's Number $R = \frac{dV}{\nu}$

$Q_{min.} = 0.002062\, d_i$ (based on $R = 3000$, below which flow is considered to be laminar)

C = Hazen-Williams constant
d_i = diameter of pipe in inches
d = diameter of pipe in feet
Q = discharge in c.f.s.
r = hydraulic radius in feet
s = slope of energy gradeline in ft. per ft.
V = velocity of flow in f.p.s.
ν = kinematic viscosity in ft.² per sec. (assumed to be 1.05×10^{-5} ft.² per sec. for water at 70°F.)

Note: Dashed lines pass through minimum values of Q/C for the given C. For lesser values of Q/C, according to the assumptions of ν and R, there can be no assurance of turbulent flow.

Example: Find size pipe required to carry $Q = 0.042$ cfs with a maximum energy gradient $S = 0.00046$ if $C = 120$. Then $Q/C = \frac{0.042}{120} = 3.5 \times 10^{-4}$. Enter chart, with above values, and find $d_i = 4$ in.. Actual head loss in 4 in. pipe $= 3.85 \times 10^{-4}$ ft. per ft.

Commercial Wrought Steel Pipe Standard Grade

Nominal diameter d_i inches	Actual diameter d_i inches	Flow area sq. ft.
1	1.049	0.00600
1½	1.610	0.01414
2	2.067	0.02330
2½	2.469	0.03325
3	3.068	0.05134
4	4.026	0.08840
5	5.047	0.1389
6	6.065	0.2006
8	7.981	0.3474
10	10.020	0.5475
12	12.000	0.7854

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-40
SHEET 1 OF 1
DATE 7-21-50

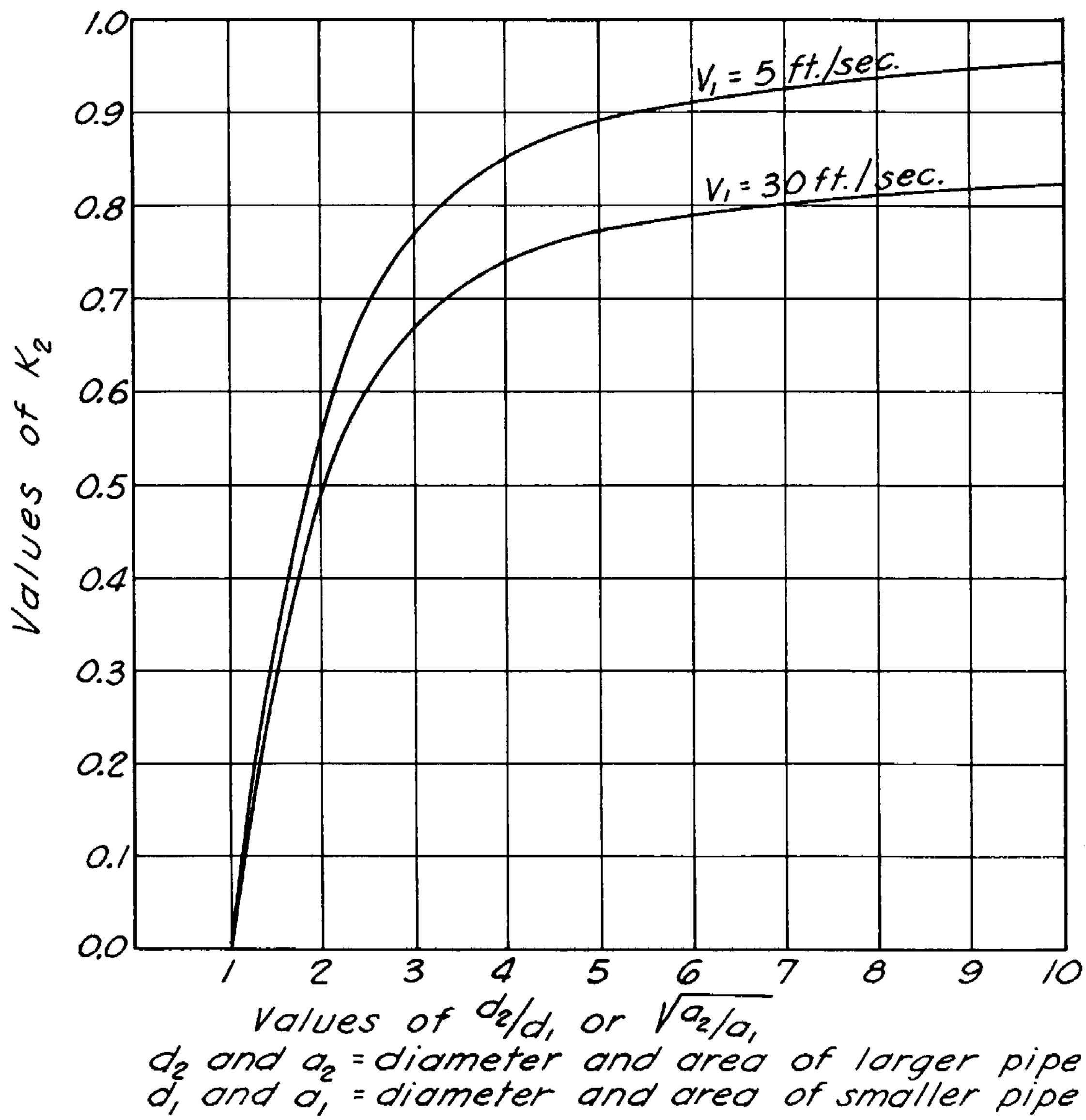
REVISED 3-7-51

5.4.2 Enlargement Loss. Loss of head due to enlargement of the pipe section may be computed by:

$$H_2 = K_2 \frac{v_1^2}{2g} \tag{5.5-18}$$

H_2 = head loss due to enlargement in ft.
K_2 = a coefficient depending on the degree of enlargement of the pipe section.
v_1 = mean velocity in the smaller pipe in fps.

Values of K_2 for sudden enlargement may be taken from fig. 5.5-2. If values corresponding more closely to given velocities are required, refer to King's Handbook, table 73, p. 231. When losses due to gradual enlargement are to be estimated, refer to King's Handbook, table 74, p. 231, for values of K_2 for conical enlargements.



LOSS COEFFICIENT FOR SUDDEN ENLARGEMENT

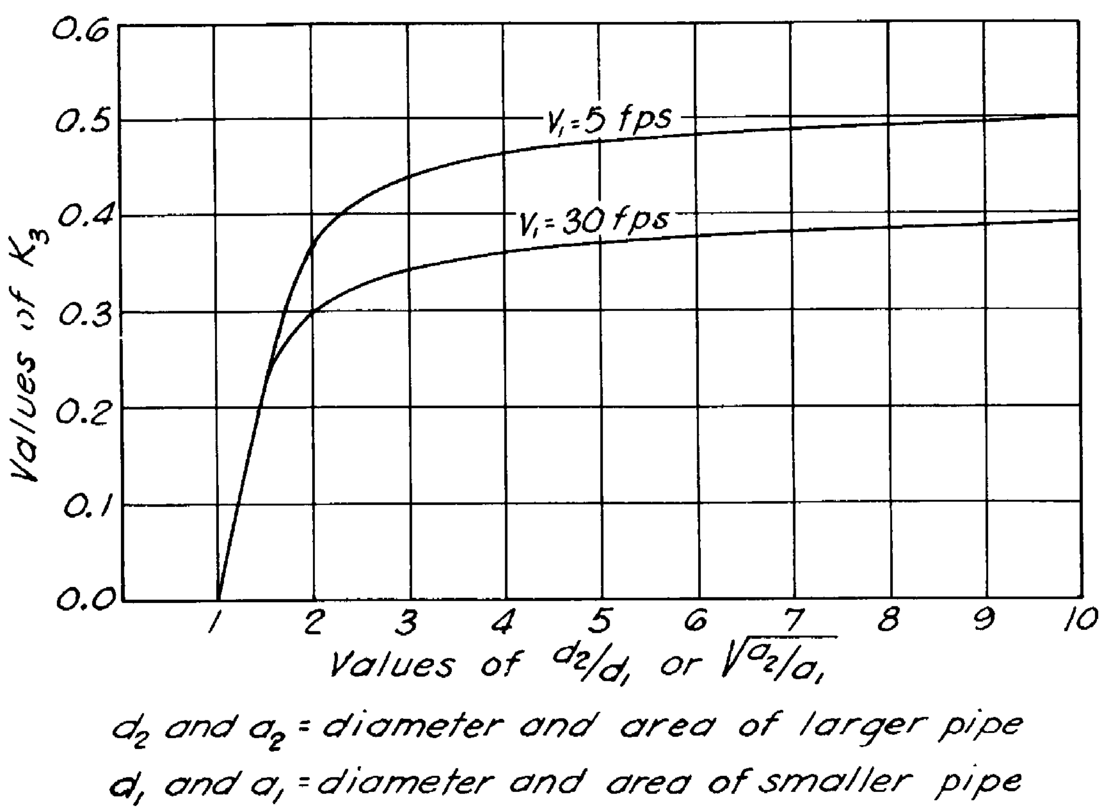
FIG. 5.5-2

5.4.3 Contraction Loss. Loss of head due to sudden contraction of the pipe section may be computed by:

$$H_3 = K_3 \frac{v_1^2}{2g} \tag{5.5-19}$$

H_3 = head loss due to sudden contraction in ft.
K_3 = a coefficient depending on the degree of reduction in the pipe section.
v_1 = mean velocity in the smaller pipe in fps.

Values of K_3 may be taken from fig. 5.5-3. Inspection of that figure will show that K_3 varies primarily with the ratio of the larger to the smaller diameter and secondarily with velocity in the smaller pipe. When it is desired to use values of K_3 corresponding more closely to given velocities than may be interpolated from fig. 5.5-3, they may be obtained from King's Handbook, table 76, p. 232. Where one or both pipes have other than a round section, convert cross-sectional area to equivalent diameter to determine the ratio of the larger to the smaller diameter; or use the square root of the ratio of the larger area to the smaller area.



LOSS COEFFICIENT FOR SUDDEN CONTRACTION

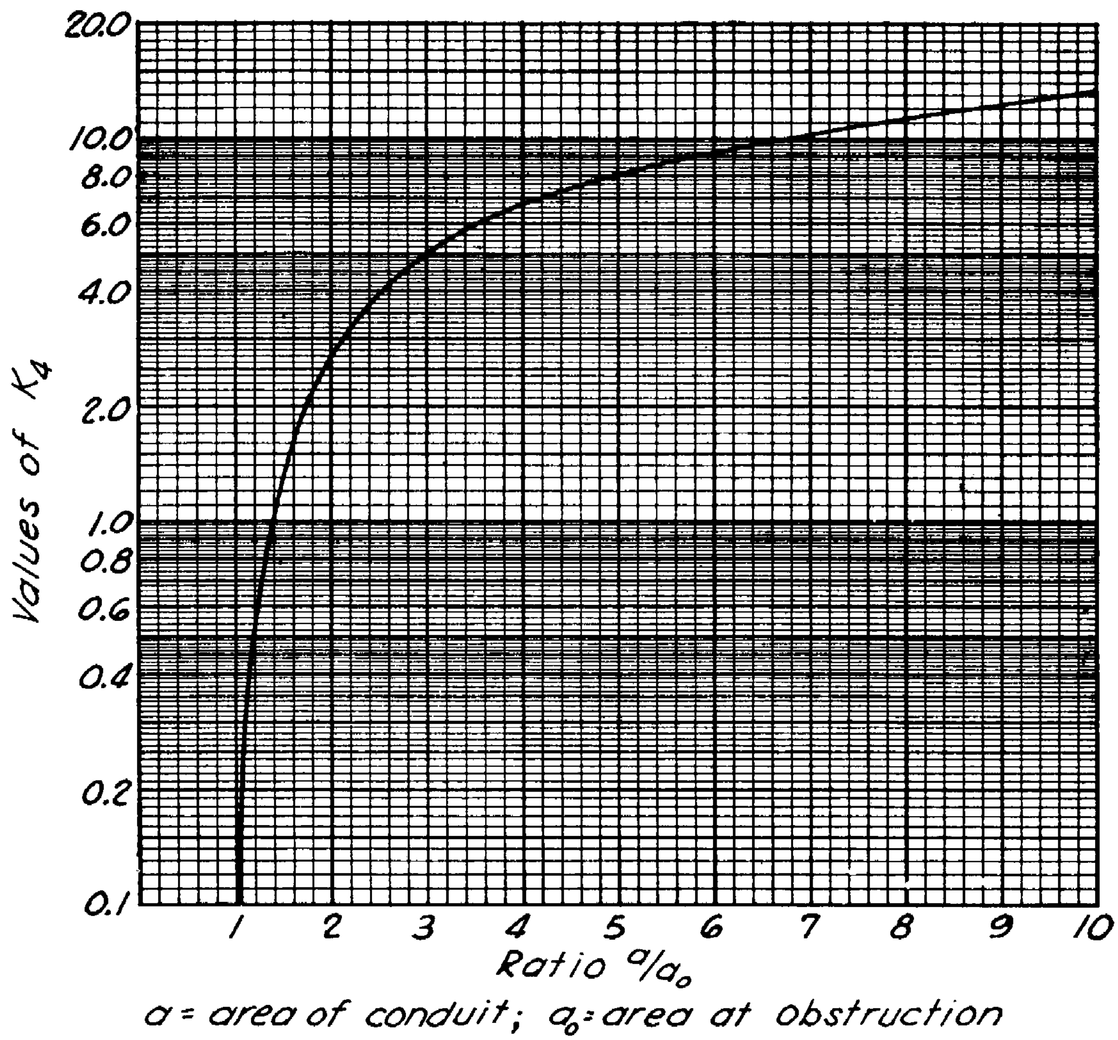
FIG. 5.5-3

5.4.4 Obstruction Loss. Loss of head due to obstruction may be computed by:

$$H_4 = K_4 \frac{v^2}{2g} \tag{5.5-20}$$

H_4 = head loss due to obstruction.
K_4 = a coefficient.
v = mean velocity in pipe.

In practice the most common types of obstructions for which head losses must be determined are valves. Generally reliable values of K_4 for any type of obstruction may be taken from fig. 5.5-4. However, careful judgment should be exercised in selecting K_4 in many cases. Where it is important that reliable determinations of head losses for valves be made, K_4 should be taken from sources of data relating to the specific type or types of valves under consideration.



LOSS COEFFICIENT FOR PIPE OBSTRUCTION

FIG. 5.5-4

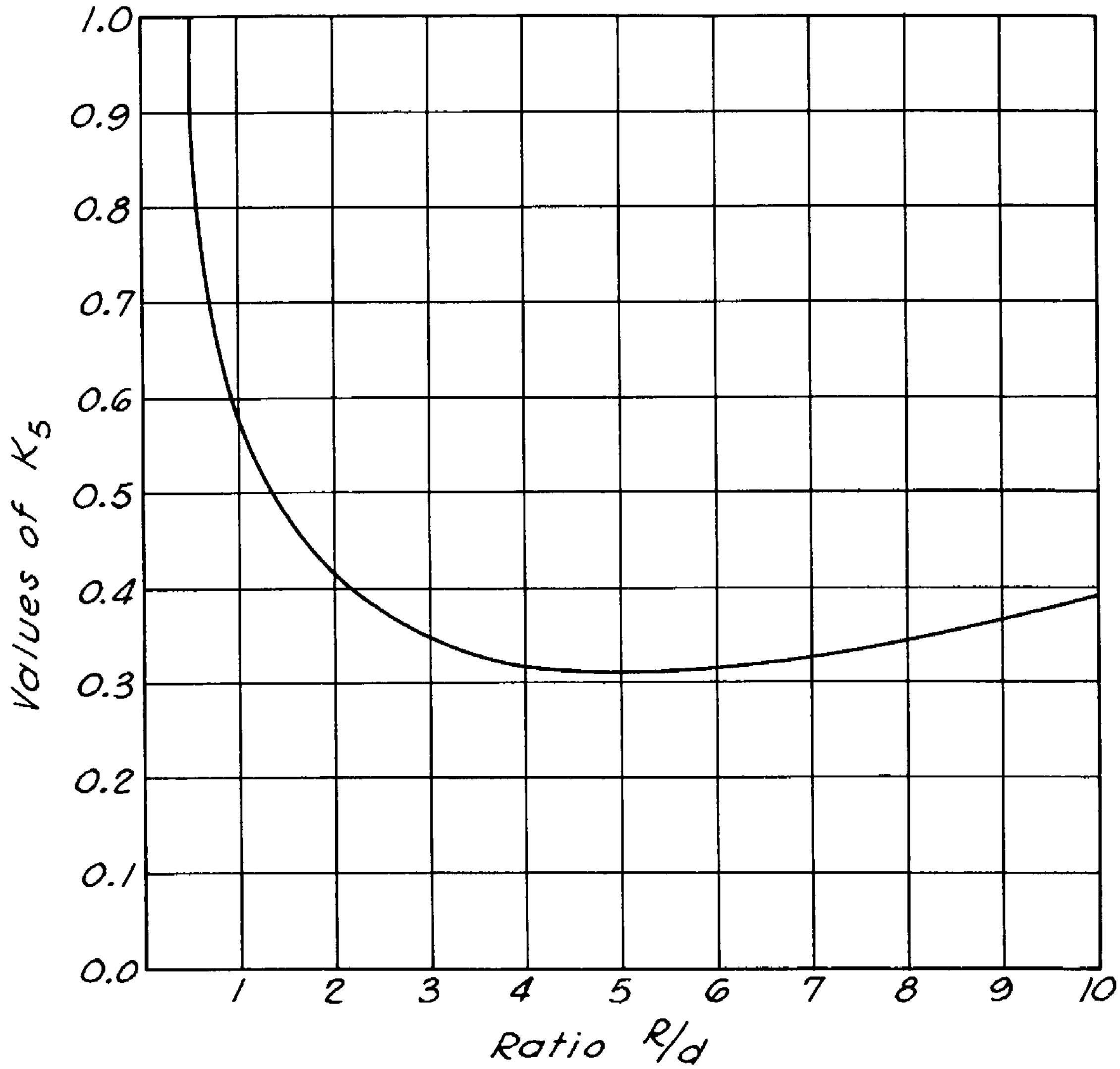
5.4.5 Bend Loss. Loss of head due to bends may be computed by:

$$H_5 = K_5 \frac{v^2}{2g} \qquad (5.5\text{-}21)$$

H_5 = head loss due to bend.
K_5 = a coefficient.
v = mean velocity.

Head loss in a pipe bend is taken as the loss in excess of the friction head. That is, the total loss in a bend is the friction head loss in an equal length of straight pipe plus the head loss due to the bend. The head loss due to the bend is computed separately.

Values of K_5 for 90-degree, curved bends that will be safe for most cases in Service work may be taken from fig. 5.5-5.



R = radius of ℄ of bend; d = diameter of circular section or side of square section

LOSS COEFFICIENT FOR 90° PIPE BEND

FIG. 5.5-5

K_5 for 90-degree square elbows, sometimes called miter bends, where there is no rounding of the corners of the intersecting conduits at either outside or inside of the bend should be taken as 1.25 to 1.50. In cases of bends where the deflection is less than 90 degrees, determine K_5 as follows:

$$K_5 \text{ (for bend} < 90^{\circ}) = \left[1 - \left(\frac{90 - \text{deflection in degrees}}{90}\right)^2\right] K_5 \text{ for } 90^{\circ} \text{ bend}$$

It is impractical to reproduce here even a limited number of the many tables, diagrams, and charts available in trade literature that give losses for standard pipe fittings, various types of valves, etc.

5.5 Analysis of Pipe Flow Problems. The basic equations for analyzing pipe flow are the energy equation which expresses Bernoulli's theorem and the continuity equation. Figure 5.5-6 shows the energy gradient and hydraulic gradient for a pipe of uniform section discharging from a reservoir.

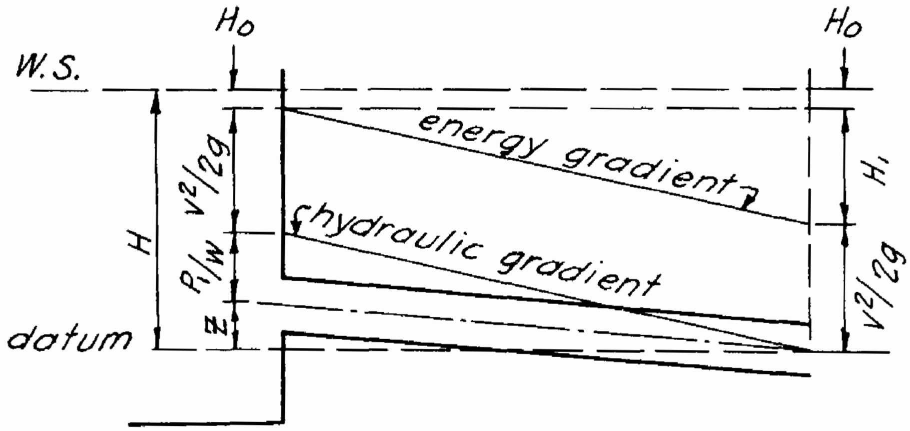


FIG. 5.5-6

The energy equation is:

$$\frac{v^2}{2g} + \frac{p_1}{w} + H_o + z = \frac{v^2}{2g} + H_o + H_1$$

and the friction head,

$$H_1 = (K_c \text{ or } K_p)\ L\ \frac{v^2}{2g} \qquad (5.5\text{-}6)$$

When H_1, d or d_1, and L are given and Q is required, v is computed by formula (5.5-6) and Q is computed by Q = av. When H_1, L and Q are given and d is required, the solution is usually made by trial and error since various types of pipe are available only in certain standard sizes. Select a trial size pipe and compute $v = Q \div a$; compute H_1 and compare with the permissible H_1; repeat trials until a standard size is found which will give the required discharge with a loss of head equal to or less than the permissible H_1.

For the complete solution of general problems of pipe flow, Bernoulli's theorem requires that the total head, H, be represented by velocity head plus all losses. In the simple case illustrated by figure 5.5-6,

$$H = \frac{v^2}{2g} + H_o + H_1$$

In the general case,

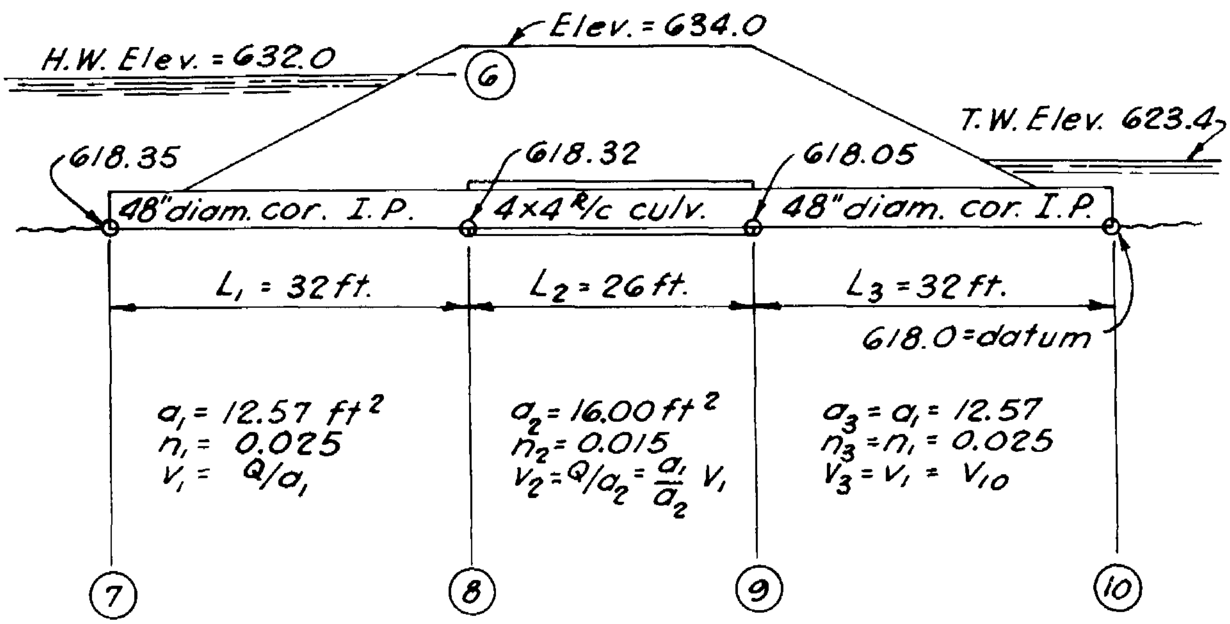
$$H = \frac{v^2}{2g} + H_1 + \text{all other losses.}$$

Methods of evaluating other losses are given in subsection 5.4.

5.5.1 Examples in Pipe Flow.

EXAMPLE 1

Given: A culvert at which the road grade is to be raised and widened and the culvert lengthened. The conditions are shown by the sketch below.



To determine: The discharge capacity of the lengthened culvert.

Solution:

The energy equation between section 6, which is horizontal and coincident with the upstream water surface, and section 10 is:

$$\frac{v_6^2}{2g} + \frac{p_6}{w} + z_6 = \frac{v_{10}^2}{2g} + \frac{p_{10}}{w} + z_{10} + K_{O7}\frac{v_1^2}{2g} + K_{p7-8}\,L_1\,\frac{v_1^2}{2g}$$

$$+ K_{28}\frac{v_1^2}{2g} + K_{c8-9}\,L_2\,\frac{v_2^2}{2g} + K_{39}\frac{v_3^2}{2g} + K_{p9-10}\,L_3\,\frac{v_3^2}{2g}$$

Each term of this equation is explained and evaluated or reduced to its simplest form in order:

$v_6^2 \div 2g$; the velocity head at section 6, which by inspection is zero.

$p_6 \div w$; the pressure head at section 6, which at the water surface is zero.

z_6; the elevation head at section 6 or the vertical distance from the assumed datum to the point of measurement of pressure head (see fig. 5.3-1), in this case 632.00 − 618.00 = 14.00 ft.

$v_{10}^2 \div 2g$; the velocity head at section 10, see data on sketch. Since $a_3 = a_1$ and $v_{10} = v_3 = v_1$, this head may be expressed as $v_1^2 \div 2g$.

$p_{10} \div w$; the pressure head at section 10, the elevation of tailwater minus the elevation of the centerline of the pipe, 623.40 − 620.00 = 3.40 ft.

z_{10}; the elevation head at section 10, the elevation of the centerline of the pipe minus the datum elevation, 620.00 − 618.00 = 2.00 ft.

$K_{o7}(v_1^2 \div 2g)$; the entrance loss at section 7. In this case the inlet is between inward projecting and sharp cornered with contraction suppressed around the part of the circumference near the bottom of the inlet and, as chosen by judgment from subsection 5.4.1, $K_o = 0.65$.

$K_{p7-8}\ L_1\ (v_1^2 \div 2g)$; the friction head loss in the corrugated pipe between sections 7 and 8. From table 5.4-1, n = 0.025 for corrugated pipe and from drawing ES-42, $K_p = 0.0182$ and $a_1 = 12.57\ ft^2$ for $d_1 = 48$ in. and n = 0.025.

$K_{28}(v_1^2 \div 2g)$; the head loss due to sudden enlargement at section 8. From the sketch $a_2 = 16.00\ ft^2$ and $a_1 = 12.57\ ft^2$, then $(a_2 \div a_1)^{0.5} = 1.27^{0.5} = 1.13$ and from figure 5.5-2, $K_2 = 0.05$. Note that K_2 is expressed in terms of the velocity head in the smaller pipe.

$K_{c8-9}\ L_2(v_2^2 \div 2g)$; the friction head loss in the concrete culvert between sections 8 and 9. From drawing ES-42, $K_c = 0.00656$ for 4x4 conduit with n = 0.015. In order to have only one unknown in the equation, v_2 is expressed in terms of v_1. Since $Q = a_1v_1 = a_2v_2$; $v_2 = v_1(a_1 \div a_2)$ and $v_2^2 = v_1^2(a_1 \div a_2)^2 = v_1^2(12.57 \div 16)^2 = 0.615\ v_1^2$.

$K_{39}(v_3^2 \div 2g)$; the head loss due to sudden contraction at section 9 expressed in terms of velocity head in the smaller pipe. See subsection 5.4.3 and figure 5.5-3. The value of $(a_2 \div a_1)^{0.5}$ at section 9 is the same as at section 8 and is equal to 1.13. From figure 5.5-3, $K_3 = 0.04$. Since $a_3 = a_1$, $v_3 = v_1$.

$K_{p9-10}\ L_3(v_3^2 \div 2g)$; the friction head loss in the corrugated pipe between sections 9 and 10. This loss is equal to and is computed in the same way as $K_{p7-8}\ L_1(v_1^2 \div 2g)$. Note that $L_3 = L_1$ and $v_3 = v_1$.

The equation, with the terms evaluated, is:

$$0 + 0 + 14.00 = \frac{v_1^2}{2g} + 3.40 + 2.00 + 0.65\frac{v_1^2}{2g} + (0.0182 \times 32)\frac{v_1^2}{2g}$$

$$+ 0.05\frac{v_1^2}{2g} + (0.00656 \times 26 \times 0.615)\frac{v_1^2}{2g} + 0.04\frac{v_1^2}{2g}$$

$$+ (0.0182 \times 32)\frac{v_1^2}{2g}$$

$$8.60 = \frac{v_1^2}{2g}\,(1 + 0.65 + 0.58 + 0.05 + 0.10 + 0.04 + 0.58)$$

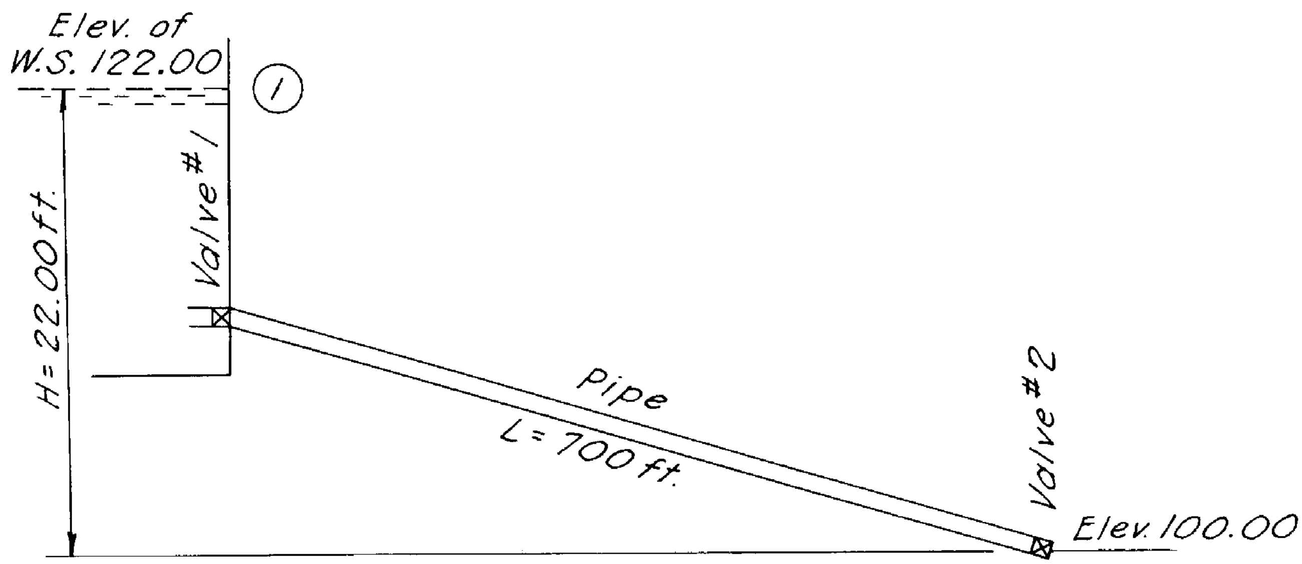
$$v_1^2 = \frac{2g \times 8.60}{3} = \frac{64.4 \times 8.60}{3} = 185$$

$$v_1 = \sqrt{185} = 13.6 \text{ fps}$$

$$Q = a_1 v_1 = 12.57 \times 13.6 = 171 \text{ cfs.}$$

EXAMPLE 2

Given: A galvanized pipe line discharging from a reservoir as shown by the sketch below. The pipe is required to deliver 15 gpm when both valves are full open.



To determine: The size of standard pipe required to deliver a minimum of 15 gpm when valve 1 and valve 2 are full open.

Solution:

1. Converting 15 gpm to cfs: $15 \times \frac{\text{gal}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ ft}^3}{7.48 \text{ gal}} = 0.033$ cfs.

2. By Bernoulli's theorem: the total head is equal to velocity head plus all head losses:

$$H = \frac{v^2}{2g} + H_o + H_{41} + H_{42} + H_1$$

H_o; the entrance loss. The inlet is inward projecting; therefore, from subsection 5.4.1, $K_o = 0.78$ and $H_o = 0.78(v^2 \div 2g)$.

H_{41} and H_{42}; the losses due to obstruction at valves. See subsection 5.4.4. When valves are full open $(a \div a_o) = 1.0$, and from figure 5.5-4, $K_4 = 0.1$. $H_{41} + H_{42} = 0.2\ (v^2 \div 2g)$.

H_1; the friction loss. From table 5.4-1, n = 0.015. By equations (5.5-4) and (5.5-6),

$$H_1 = K_p L \frac{v^2}{2g} \quad \text{and} \quad K_p = \frac{5087\ n^2}{d_1^{4/3}}$$

In this case,

$$H_1 = 700 \times \frac{5087 \times (0.015)^2}{d_i^{4/3}} \times \frac{v^2}{2g} = \frac{700 \times 1.14}{d_i^{4/3}} \times \frac{v^2}{2g} = \frac{800}{d_i^{4/3}} \times \frac{v^2}{2g}$$

3. Placing the evaluated terms in the energy equation:

$$22.00 = \frac{v^2}{2g} + 0.78\frac{v^2}{2g} + 0.20\frac{v^2}{2g} + \frac{800}{d_i^{4/3}} \times \frac{v^2}{2g}$$

$$22.00 = \frac{v^2}{2g}\left(1.98 + \frac{800}{d_i^{4/3}}\right)$$

4. Solving for d_i by trial using the continuity equation to compute v for the assumed pipe sizes when Q = 0.033 cfs gives the smallest standard pipe that will deliver 0.033 cfs.

Try d_i = 2 in; $a = 0.023$ ft^2; $v = \frac{Q}{a} = \frac{0.033}{0.023} = 1.44$ fps

$$\frac{v^2}{2g} = 0.032; \quad 2^{4/3} = 2.52; \quad \frac{800}{2.52} = 317$$

$$0.032(1.98 + 317) = 10.2 \text{ ft.}$$

Try d_i = 1-1/2 in; $a = 0.014$ ft^2; $v = \frac{0.033}{0.014} = 2.36$ fps

$$\frac{v^2}{2g} = 0.087; \quad 1.5^{4/3} = 1.72; \quad \frac{800}{1.72} = 465$$

$$0.087(1.98 + 465) = 40.7 \text{ ft.}$$

2-inch pipe must be used since a total head of 40.7 ft would be required to produce Q = 0.033 cfs through the 1-1/2 inch pipe.

Discharge for the 2-inch pipe under a head of 22.00 ft is:

$$22.00 = \frac{v^2}{2g}(1.98 + 317) = \frac{v^2}{2g}(318.98)$$

$$v = \sqrt{\frac{64.4 \times 22.00}{318.98}} = 2.10 \text{ fps}$$

$$Q = av = 0.023 \times 2.10 = 0.048 \text{ cfs.}$$

Alternate solution:

1. In step 3 of the above solution it is shown that entrance loss plus obstruction loss at the valves amounts to only $(0.78 + 0.20)(v^2 \div 2g)$ while friction loss is $(800 \div d_i^{4/3})(v^2 \div 2g)$; therefore, losses other than friction are negligible. The slope of the hydraulic gradient may be taken as $s = 22.00 \div 700 = 0.0314 = 3.1 \times 10^{-2}$.

2. From table 5.5-2, C for small sizes of galvanized pipe is found to be 80, then $Q \div C = 0.033 \div 80 = 0.00041 = 4.1 \times 10^{-4}$.

3. On drawing ES-40 the point where $s = 3.1 \times 10^{-2}$ and $Q \div C = 4.1 \times 10^{-4}$ falls between the lines for $d_1 = 2$ in. and $d_1 = 1.5$ in. The larger size, i.e., 2-inch, standard pipe is required.