Simplified Method of Predicting Fall of Water Table in Drained Land

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THIS paper presents a simplified procedure for predicting rate of fall of the water table in tile-drained or ditch-drained land. The procedure is based on steady-state theory and abrupt drainage of pore space. In this respect, it lacks the theoretical sophistication of certain other treatments (2, 4, 5, 7, 8, 13)*. Its simplicity, general applicability, and apparent accuracy, however, favor use of the proposed procedure in routine drainage design. The first part of the paper presents the simplified design procedure, which is discussed and compared with other solutions.

Procedure

Principles

The procedure relates to the fall of the water table midway between drains, where the water table recession is the slowest and, therefore, the most critical. If the water table is assumed to fall without change of shape, the flux per unit area of water table is uniform between drains. Therefore, steady-state drainage relationships, which also assume a uniform flux, can be used to describe the rate of fall (dm/dt) of the water table midway between the drains, or

$$\frac{dm}{dt} = -\frac{P}{f} \quad . \; . \; . \; . \; . \quad [1]$$

where

P = instantaneous drainage rate or coefficient (dimension length/time)

f = drainable porosity (dimensionless)

and m = height of water table midway between drains above tile center (Fig. 1).

The assumption underlying equation [1] is that the instantaneous drainage rate midway between drains can be taken as equal to the steady-state drainage rate corresponding to the same

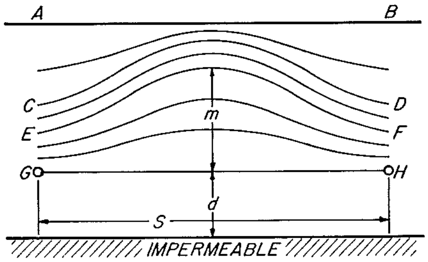

FIG. 1 Geometry and symbols for falling water table in drained land.

value of m. The minus sign in equation [1] accounts for m decreasing with increasing t. The procedure in this paper can also be applied to rising water tables. In that case, the minus sign in equation [1] is omitted.

The principle for predicting the rate of fall of the water table midway between drains consists essentially of integrating equation [1] to obtain a relationship between m and t. The previously stated assumption implies that the P *versus* m relationship utilized in this integration can be taken from existing steady-state solution methods for tile or ditch drainage as the case may be.

The assumption that the water table falls without change of shape is of limited validity. For some time following a ponded condition, the water table falls faster near the drains than midway between the drains and the average flux P per unit area of water table exceeds the flux $f\ dm/dt$ midway between drains. The region where this condition applies is shown as zone ABCD in Fig. 1, which is schematically taken from drawings by Kirkham and Gaskell (8) and Isherwood (7). After having receded faster near the drains than midway between the drains, the water table reaches a position where it falls for some time without appreciable change in shape (zone CDEF in Fig. 1) and P equals $f\ dm/dt$. As recession progresses, the water table eventually falls faster midway between the drains than in the vicinity of the drains (zone EFGH in Fig. 1) and P is less than $f\ dm/dt$.

Thus with a falling water table the flux in general varies with distance from the drain. Hence, in order to use steady-state solutions, which assume a flux that is independent of time and of distance from the drains, for prediction of the rate of fall of the water table, a correction factor C is introduced in equation [1], yielding

$$P = -fC\frac{dm}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

According to the previous paragraph, C can have the following ranges:

$C > 1$ for region ABCD
$C = 1$ for region CDEF
$C < 1$ for region EFGH

Generally C appears to be between 0.8 and 1.0, except for the first stages of recession following a ponded case where C is higher. A more detailed discussion on selecting C is presented in the section, entitled "The factor C."

Equation [2] can be integrated analytically or numerically, depending on whether the relationship between P and m is available in terms of an equation or in tabular or graphical form. Both the analytical and numerical integration will be discussed in the following paragraphs.

Analytical Solution

The analytical solution utilizes steady-state drainage formulas where P is expressed as a function of hydraulic conductivity K, drain spacing S, height m of water table midway between drains above the level of the drains, depth d of impermeable material below level of the drains, and drain radius (See Fig. 1 for geometry and symbols). The equation selected in this paper for the integration of equation [2] is Hooghoudt's equation, written here as

$$P = \frac{4Km(2d_e + m)}{S^2} \quad . \; . \; . \; . \; . \quad [3]$$

where d_e is the "equivalent layer" to take into account the flow convergence near the drains (6). Tables are presented by Hooghoudt (6) which show d_e as a function of d, S, and r. A graph by van Schilfgaarde (13), giving Hooghoudt's d_e values in feet as a function of d and S for a tile diameter of 5 in., has been expanded to cover a wider range of spacings and is shown here as Fig. 2.

Substituting equation [3] into equation [2], integrating between $t = 0$, m_o and t, m_t with C considered constant, and rearranging terms gives

$$\frac{Kt}{f} = \frac{2.3CS^2}{8d_e}\log_{10}\frac{m_o\ (m_t + 2d_e)}{m_t\ (m_o + 2d_e)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

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* Numbers in parentheses refer to the appended references.

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Equation [4] can be used to calculate Kt/f for a selected water table drop from m_o to m_t and a certain S and d_e. The time for the water table to drop from m_o to m_t is then computed by multiplying Kt/f by f/K for the particular soil. For example, if $K = 7.5$ ft per day; $f = 0.15$; $d = 5$ ft; $S = 125$ ft, and $C = 0.8$ (see section "The factor C"), Kt/f for a water table drop from $m = 6$ ft to $m = 5$ ft is computed with equation [4] as 41 ft, which gives $t = 0.8$ day. The d_e value for this computation was evaluated as 4.3 ft from Fig. 2.

To find a drain spacing that gives a specified rate of water table drop, assumed S values are used in Equation [4] until the desired Kt/f is obtained.

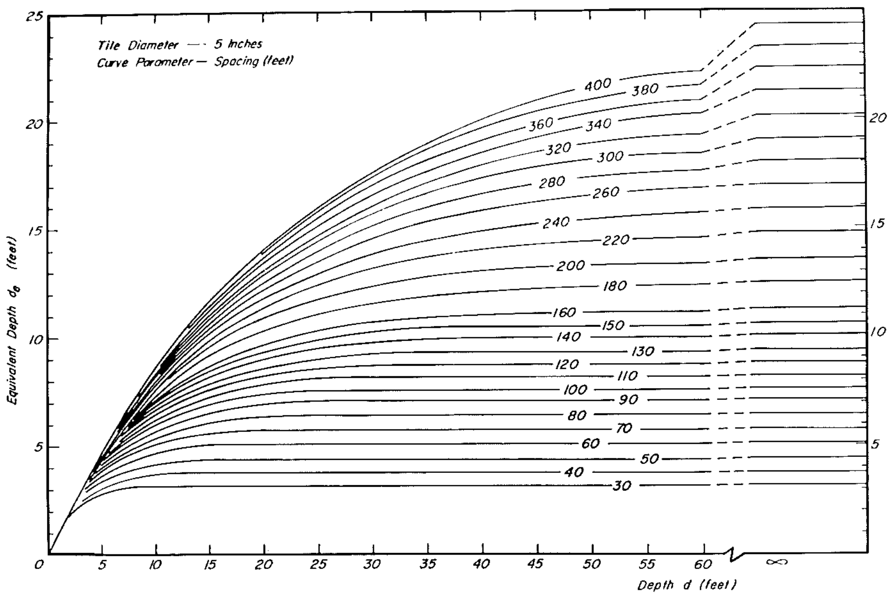

FIG. 2 Hooghoudt's equivalent depth d_e as a function of d for different drain spacings (on the curves).

Numerical Solution

The numerical solution of equation [2] utilizes relationships between P and m in graphical or tabular form. The solution is, therefore, applicable to any solution method of the steady-state condition, whether it is analytical, numerical, via nomographs, analogs, models, field experiments, or others. The numerical integration is carried out with a plot of P as a function of m for a given drain design (hypothetical example shown in Fig. 3). The m scale is divided into increments Δm. For each Δm, the mean drainage rate $\bar{P}$ is determined. The time Δt for the water table to drop an increment Δm is then simply computed as

$$\Delta t = \frac{Cf\Delta m}{\bar{P}} \quad . \; . \; . \; . \; . \; . \; . \quad [5]$$

The total time required for a water table drop from m_o to m_t is determined by adding the increments Δt for the Δm increments between m_o and m_t. This procedure also lends itself for taking into account variable drainage porosity with depth, as illustrated in the example in Table 1, which applies to the P-m relation of Fig. 3. If the problem is to find a drain spacing that meets a specified rate of fall of the water table, the procedure in Table 1 is repeated with P-m curves for different S values until the desired spacing is found.

Direct Numerical Solution for Drain Spacing

A more general approach that eliminates the trial-and-error procedure for finding the desired drain spacing, utilizes P versus m curves for a number of drain spacings. An example of such a family of curves is shown in Fig. 4, where P/K is plotted against m/d for different values of S/d. This graph was obtained from the nomographs, which were developed by Ernst and Boumans (12, pp. 93 and 94) in The Netherlands, where tile diameters of 2 to 3 in. are commonly used. In using plots as Fig. 4, the tile depth must be selected first so that d can be evaluated. A certain distance of fall of the water table from m_o to m_t is then selected and the corresponding m_o/d and m_t/d are marked on the graph. From the specified time Δt for the water table to fall from m_o to m_t, $\bar{P}$ is calculated with equation [5] as

$$\bar{P} = \frac{Cf\Delta m}{\Delta t} \quad . \; . \; . \; . \; . \; . \; . \quad [6]$$

Knowing the hydraulic conductivity of the soil, $\bar{P}/K$ is now computed. The S/d curve for which the average P/K between m_o/d and m_t/d is equal to the computed value of $\bar{P}/K$ is determined from Fig. 4. Since d is known, the spacing S can be calculated.

To illustrate this procedure, it will be applied to the same conditions as the example presented under the section "Analytical Solution." To this end, the spacing giving the same rate of water-table drop, that is, from $m = 6$ ft to $m = 5$ ft in 0.8 day, will be determined. Taking $C = 0.8$, equation [6] yields $\bar{P} = 0.15$ ft per day, which gives $\bar{P}/K = 0.02$. The m/d values corresponding to $m = 6$ ft and $m = 5$ ft are, respectively, 1.2 and 1.0. The broken lines in Fig. 4 show that the curve giving an average value of P/K of 0.02 between m/d values of 1.0 and 1.2 has an S/d value of 25. Since $d = 5$ ft, the required spacing is 125 ft. This equals the spacing used in the previous example for the analytical solution.

The factor C

The factor C in equation [2] can be considered the ratio of the average flux between drains to the flux midway between the drains. In theory, this is valid, only if the average of a non-uniform flux gives the same drainage flow as when that average was uniformly distributed along the water table, which can be expected to be true for relatively low degrees of non-uniformity of flux. In that case, C can be determined as the ratio of the average distance of fall of the entire water table to the fall midway between the drains for a certain time increment. Bouwer (1) has shown that this ratio is about 0.8 for m/S values from 0.02 to 0.08 and low d values. A C value of approximately 1.0 is indicated by Childs' work (3) for higher water tables with m/S exceeding 0.15. The drainage literature abounds in other examples of water tables receding with little change in shape, all pointing to C values of approximately 1.0. C values in excess of 1.0 can be expected for the initial stages of water-table recession following a ponded case.

For the drop immediately following the ponded case, C will be high initially

TABLE 1. EXAMPLE OF NUMERICAL SOLUTION FOR CALCULATING RATE OF FALL OF WATER TABLE
(For curve in Fig. 3, C = 1)

m, feet	Δm, feet	f	fΔm, feet	$\bar{P}$, ft/day	Δt, days	t, days
5						0
	1	0.2	0.2	0.165	1.2	
4						1.2
	1	0.15	0.15	0.125	1.2	
3						2.4
	1	0.15	0.15	0.088	1.7	
2						4.1

but will undergo a rapid reduction as the rate of fall of the water table near the drains becomes slower. Thus, C values for the first stages of water-table recession following a ponded condition cannot be obtained very well, and the use of a constant C in the integration is objectionable. However, the Kt/f values for subsequent stages of water-table recession are much larger than for the first increment of water-table drop, so that the relative effect of errors in the initial C values on the Kt/f values for the more advanced stages of water table recession is small.

As an illustration, equation [4] was applied to calculate Kt/f for a falling water-table analysis by Kirkham and Gaskell (8). In that analysis [Fig. 4 in (8)], water-tables at Kt/f of 2 ft, 4 ft, and 6 ft are shown for a receding water-table in tile-drained land from a ponded condition at $Kt/f = 0$. C values for the three water-table positions were determined by measuring the area between field surface and the successive water-tables with a planimeter. The average distance of fall of the water-table was then computed and divided by the distance of fall midway between the drains to yield C. The resulting C values for the Kt/f values of 2 ft, 4 ft, and 6 ft were 2.9, 2.7, and 2.3, respectively. Substituting these C values in equation [4] gave Kt/f values of 2.7 ft, 4.8 ft and 6.8 ft. Thus, the relative error was largest for the first increment of water-table recession, which can probably be attributed to the large variation in C for the first water-table drop following a ponded condition. The calculated Kt/f values for the subsequent water table positions, 4.8 ft and 6.8 ft, respectively, show better agreement with the values of 4 ft and 6 ft given by Kirkham and Gaskell. The Kt/f increments for the second and third increments of water-table drop resulting from the calculated values with equation [4], *i.e.*, 2.1 and 2.0 ft, respecdively, are almost the same as the theoretical Kt/f increments of 2 ft given by Kirkham and Gaskell.

To illustrate the small relative effect of erroneous C values for the first increment of water table drop on the subsequent Kt/f values, Kt/f for a recession from the ponded case to $m = 1$ ft will be computed for the same case [Fig. 4 in (8)]. Starting with the water-table at $Kt/f = 6$ ft and assuming a C of 0.80 for the water-table recession thereafter, the additional Kt/f for the water table to drop to $m = 1$ ft is calculated with equation [4] as 24 ft. The total Kt/f for a water table recession from the ponded case to $m = 1$ ft is, therefore, equal to $6 + 24 = 30$ ft. Calculating Kt/f for the same total drop with equation [4], using a C value of one, yields 33 ft, which does

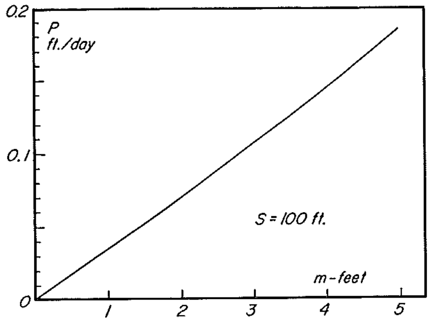

FIG. 3 Example of P versus m relationship for graphical integration of equation [2].

not differ much from the 30 ft previously obtained. The C value of 1.0 was selected in this calculation of Kt/f, because short-term C values in excess of, equal to, and less than 1.0, could be expected for this recession.

From this discussion, it is apparent that C can generally be selected as unity. For relatively low water tables with $(m/S) < 0.1$, C can be taken as 0.8. For high water tables near the drains that recede much faster than the water table midway between drains, such as following a ponded case, C values in excess of 1.0 must be selected, for instance, from 2 to 3. The application of the procedures in this paper to the initial stages of water-table recession after a ponded condition, however, must be considered with reservation.

Capillary Fringe

The role of the capillary fringe in falling water-table analyses can be summarized as follows:

(a) The fringe provides an extra path for the drainage flow, so that P for a certain m is higher with fringe than without fringe. Bouwer (1) has shown that this effect can be accounted for by adding the fringe thickness to d in the evaluation of d_e.

(b) Drainage of pore space takes place only if the distance of the water table below field surface exceeds the fringe thickness. Thus very rapid rates of water-table recession occur if the water table is sufficiently high for the fringe to extend to field surface (2).

(c) From a soil-aeration standpoint, the top of the capillary fringe is more significant than the water table, which is only a pressure contour. Adequate desaturation can be effected by proper selection of m_o and m_t in specifying rates of water-table recession for design purposes. Higher water tables can be tolerated in light-textured soils than in heavy-textured soils.

Steady State Solutions

Although this paper refers mainly to designing drainage systems for a certain rate of fall of the water table, the material presented will also be useful if drain spacings are to be calculated on the basis of the steady state. The problem in that case is generally to find the drain spacing that gives the desired drainage rate P at the maximum permissible water table. The latter is characterized by m after the drain depth has been selected. With Hooghoudt's equation (3), this requires a trial-and-error procedure whereby (a) a certain tile depth is selected, (b) the resulting d and m are determined, (c) a number of S values are assumed, (d) the corresponding d_e values are evaluated from Fig. 2, and (e) P is calculated with equation [3]. This procedure is repeated until S giving the desired P is found. Direct solution of S can be obtained with Fig. 4, where the S/d curve giving the desired P/K for the m/d value in question, can be directly evaluated.

Effect of Drain Diameter

According to equation [3], the effect of drain diameter on drain spacing is proportional to the effect of drain size on $\sqrt{2d_e + m}$. Hooghoudt's tables (6) show that the effect of tile diameter on d_e is small and increases with increasing d, as illustrated in Table 2. Since S varies with even less than $\sqrt{d_e}$, the effect of tile size on S is relatively small and Figs. 2 and 4 can be used for other drain diameters as well.

TABLE 2. EFFECT OF TILE DIAMETER (OD) ON d_e FOR TWO VALUES OF d AND S = 100 FT

Tile diameter in.	d_e in feet d = 3.3 ft	d = ∞
2.4	2.79	6.70
4.7	2.95	7.65
7.9	3.05	8.46
15.8	3.21	10.00

Discussion

The integrated form of steady-state relationships for application to transient conditions is subject to the same limitations as the original steady-state relations. A number of drainage theories have been based upon the Dupuit-Forchheimer assumption of horizontal flow. The neglect by such theories of the effect of convergence of flow towards the drain causes an increasingly large error for increasing values of d. This limitation can be overcome by substituting for d an equivalent depth value, d_e, as was done by Hooghoudt (6) in equation [3].

The principle of integrating a steady-state solution for the transient case can be applied to analytical equations other than Hooghoudt's. For example, Toksöz and Kirkham (10) gave the equation.

$$m = \frac{SP}{K}\frac{1}{1-\frac{P}{K}}F(r/S,\ d/S) \quad [7]$$

where the function F is evaluated from graphs or tables. When P/K is small compared to unity, equation [7] can be written as

$$P = \frac{mK}{S\ F(r/S,\ d/S)} \quad .\ .\ .\ . \quad [8]$$

Combining equation [8] with equation [2] and integrating gives

$$\frac{Kt}{f} = CS\left\{1n\frac{m_o}{m_t}\right\}F(r/S,\ d/S). \quad [9]$$

Since the equation by Toksöz and Kirkham was developed from potential theory and not from the Dupuit-Forchheimer assumption, there is no restriction as to the magnitude of d.

To compare the integrated steady-state equations with equations that have been specifically developed for transient drainage conditions, the authors have selected the Glover equation (4), a proposal by Luthin and Worstell (9), data presented by Isherwood (7), and a more recent proposal by van Schilfgaarde (13).

The Glover equation reads:

$$S = \pi\left[\frac{Kt}{f}\left\{d + \frac{m_o}{2}\right\}/1n\frac{4\ m_o}{\pi\ m_t}\right]^{1/2}$$

It is derived from the Dupuit-Forchheimer assumption, with the additional simplification that the thickness of the water-bearing aquifer can be approximated as the constant $d + m_o/2$. Thus it neglects the effect of convergence of flow toward the drains, and this factor would lead one to expect its predictions to be increasingly less accurate as d increases. Furthermore, at small values of d, the assumption of a time-constant thickness of the aquifer is not met even approximately.

Luthin and Worstell's equation is

$$S = \frac{4\ cKt}{\pi f\ 1n(m_o/m_t)} \quad .\ .\ .\ . \quad [10]$$

The main assumption in its derivation is that the discharge rate is proportional to the height of the water table, or $q = cKm$, where c is assumed constant. It appears, however, that in fact

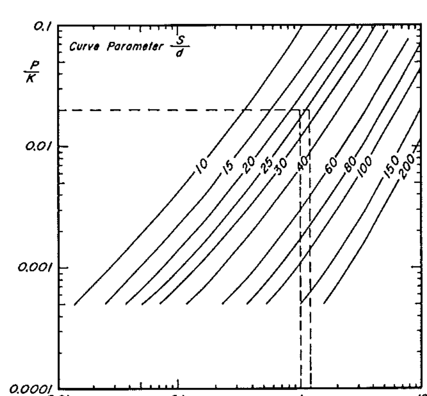

FIG. 4 Family of curves of P/K versus m/d for different S/d values for direct evaluation of desired drain spacing (from nomographs by Ernst and Boumans).

TABLE 3. COMPARISON OF FALLING-WATER TABLE PREDICTIONS
m_o = 3 ft, m_t = 2 ft, and tile radius, 3 in.

S, feet	d, feet	Kt/f in feet: Integrated Hooghoudt	Integrated Toksoz & Kirkham	van Schilfgaarde	Luthin	Glover	Isherwood
30	0	30	....	33	8.0	39	
	2	13	21	14	8.0	17	14
	4	9.6	15	10	8.0	11	11
	8	8.3	12	8.8	8.0	6.2	
	16	8.2	11	8.8	8.0	3.3	13
	32	8.2	11	8.8	8.0	1.8	
		8.2	11	8.8	8.0	0	
60	0	120		133	16	157	
	2	47	79	52	16	67	46
	4	34	47	36	16	43	44
	8	26	32	27	16	25	
	16	23	27	24	16	13	47
	32	23	26	24	16	7.1	
		23	26	24	16	0	
120	0	480		533	32	628	
	2	183	300	200	32	270	163
	4	124	167	133	32	172	190
	8	84	103	85	32	79	
	16	65	74	67	32	54	190
	32	58	64	60	32	28	
		58	62	60	32	0	
240	0	1920		2133	64	2524	
	2	731		796	64	1080	
	4	475	625	510	64	688	
	8	299	350	309	64	398	
	16	197	218	208	64	216	
	32	151	163	157	64	113	
		137	137	141	64	0	

c is a function of the drain spacing. Since the discharge rate, q, is proportional to the hydraulic gradient, and the gradient in turn is inversely proportional to the length of the average flow path, the relation for q would be better written

$$q = c_1\ K\ m/S$$

which gives $c = c_1/S$. Thus Luthin and Worstell's equation can be expected to predict too small a change in t for a given change in S. In addition, Luthin and Worstell's equation does not take into account the depth of an impervious layer. Hooghoudt (6) and van Schilfgaarde *et al* (12) have, among others, shown this to be an important factor.

The comparison with Isherwood's data is based on table 1 of his publication (7), which presents results of falling-water-table analyses with a digital computer. Some of the data in Isherwood's table show irregular trends when plotted against d.

The third transient equation with which a comparison will be made is by van Schilfgaarde (13). This equation is

$$S = 3A\left[\frac{Kt}{f}\frac{(m_t + d_e)\ \ (m_o + d_e)}{2\ (m_o - m_t)}\right]^{1/2}$$

where A represents an incomplete beta function. It is evaluated as a function of $d_e/(d_e + m_o)$ elsewhere (13), but may be approximated within 3 percent error as

$$A = (1 - [d_e/(d_e + m_o)]^2)^{1/2}$$

The derivation of van Schilfgaarde's equation is based on the Dupuit-Forchheimer assumption, but the convergence of flow is accounted for by substituting d_e for d and the thickness of the aquifer is treated as a variable, rather than as a constant as did Glover.

Table 3 summarizes a comparison of all of the solutions discussed. For each solution, the length of time required for the water table to drop one foot from an initial height of three feet above the drains was calculated. To eliminate the effect of the soil characteristics, the results were expressed in terms of Kt/f, in feet, rather than directly in days or hours. Because of the low m/S values, the C value in equations [4] and [9] was selected as 0.8. The constant c in equation [10] was taken as 1.2, to correspond with the value c = 0.1 foot per inch suggested by Luthin (9, p. 44).

The comparison shows close agreement between the integrated Hooghoudt equation and the van Schilfgaarde equation. Although the analyses leading to these solutions differed considerably, both were based on the Dupuit-Forchheimer assumptions with convergence correction through use of d_e. These two solutions also agree reasonably well with the modified Toksöz-Kirkham equation which avoids the Dupuit-Forchheimer assumptions. This mutual agreement lends support to the use of the corrected Dupuit-Forchheimer assumptions as well as to the technique of integrating the steady-state equations.

Far poorer agreement is found with the Glover equation, which appears to be too dependent on the value of d. The Kt/f values are affected by an increase in d even when d is larger than $S/5$, where d_e is no longer affected significantly by d (Fig. 2). The absurd value of $Kt/f = 0$ for $d = \infty$ is a direct consequence of the assumption upon which the equation is based.

Luthin and Worstell's equation also failed to render a realistic solution. Both the failure of the equation to account for the effect of the depth to an impervious layer and the underestimated effect of spacing can be observed from the tabulation.

The agreement with Isherwood's data is good for the lowest d value. Poor agreement at the higher d values may be due to irregular trends in Isherwood's results.

From the above comparison, it is concluded that the integrated Toksöz-Kirkham equation, the integrated Hooghoudt equation and the van Schilfgaarde equation all give consistent results which are probably accurate enough for field use. Use of either the Glover equation or the Luthin and Worstell equation is likely to result in sizable errors.

The evidence in support of the technique of integration of steady-state equations as used here also lends support to the validity of the proposed numerical integration. In essence, it accomplishes graphically the same purpose as does the analytical integration. It has the advantage that it enables one to use as a basis data on the relation between P and m obtained from model studies, relaxation or iteration calculations; as well as from analytical formulations. By using a family of such curves, the trial-and-error procedure for evaluating S is eliminated and S can be determined directly.

Summary

Steady-state drainage solutions are analytically or graphically integrated to predict the rate of fall (or rise) of the water table midway between tile lines or ditches. The procedure assumes abrupt drainage of pore space. A factor C is introduced to account for the non-uniformity of flux per unit area of water table if the water table changes in shape during recession. The procedure is amenable to steady-state formulas or to graphical or tabular solutions obtained by analogs, models, numerical procedures, field experiments, or others.

Two integrated steady-state drainage equations (by Hooghoudt and by Toksöz and Kirkham) are compared with four solutions developed specifically for the falling water table. The transient formula by van Schilfgaarde accounts both for flow convergence and changing height of the flow system and shows good agreement with the two integrated equations, which also agree mutually. The other three transient solutions have some inherent weaknesses and show poor agreement.

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