Using Mannings Equation with Natural Streams

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Background

These days almost everybody knows that straightening a stream helps neither nature nor humans. The channel is only one aspect of a riverine ecosystem. Streams create riparian zones and corridors all tied together with not only the surface water flow, but also groundwater, vegetation, wildlife, and human activity. We have competing human interests. We want to use the good land near streams for agricultural purposes, without harming the healthy balance of the stream’s sediment transport, plant life, and wildlife habitat. We need to bridge them for traffic and commerce. Healthy streams are a joy to be around. People want access for fishing, hiking, bird-watching, and overall enjoyment of nature. We like to build our homes near them. We like our restaurant views to see them.

Projects involving streams need the cooperative expertise of numerous disciplines. Engineering is very important because an appreciation of the physical power of flowing water forms the foundation for everything else. But how much detailed engineering work is necessary for a given project?

Work in the stream corridor often does not require extensive hydraulic modeling. Uniform flow calculations and the associated hydraulic parameters often suffice. However, the interdisciplinary nature of stream work can lead to technical misunderstandings by stream team members. Does Mannings equation correctly calculate hydraulic parameters for real rivers or not? The question itself indicates a less than full appreciation of the hydraulics of natural streams.

Mannings equation computes a uniform flow or normal depth approximation. The formula can come close to reality only to the extent that the real stream condition is uniform flow or normal depth. Natural streams often do not exhibit that behavior, which will be further explained below.
A common form of Mannings equation solves for flow velocity. Here, the variable R is hydraulic radius, defined as the flow area divided by the length of the cross-section wetted perimeter. The variable S is longitudinal channel bed slope. The variable n is the empirically derived roughness or boundary resistance coefficient called Manning’s roughness or n-value. Using the flow continuity equation, in which streamflow is equal to flow area times flow velocity, a second form of Mannings equation is possible, enabling a solution for flow (Q) in cubic feet per second.

\[ V = \frac{1.486}{n} R^{2/3} S^{1/2} \]

\[ Q = \frac{1.486}{n} AR^{2/3} S^{1/2} \]

These conditions are covered in basic hydraulics textbooks, such as Chow’s *Open-Channel Hydraulics* (Chow, 1959). Herein, a succinct explanation will be provided and tips will be given so that practitioners can more easily estimate how closely a stream may be expected to flow at normal depth or the Mannings equation solution.

In addition, the equation is strongly dependent on choices of roughness coefficient. Most real stream cross-sections have significantly varying boundary roughness, so that no practicioner could be expected to select an overall cross-section roughness coefficient from field observation. A better practice is to select different roughness values for different sections of the cross-section wetted perimeter. Then a composite roughness for the entire cross-section may be calculated from formulas such as given in Chow (1959) and adjusted for other conditions in the stream reach.

**Explanations**

Considering a short stream reach, the term *uniform flow* refers to a condition in which the flow depth and area, average velocity and discharge do not change from one cross-section to another in the reach, and the slopes of the channel bed, water surface, and energy gradeline are identical (thus parallel). The word “normal” in *normal depth* is meant in the mathematical sense. If the slopes of the channel bottom and the water surface profile are parallel, then the water surface is normal or perpendicular to its depth.

How often does this condition occur in natural streams? Here’s Chow (1959, page 89):

In natural streams, even steady uniform flow is rare, for rivers and streams in natural states scarcely ever experience a strict uniform-flow condition. Despite this deviation from the truth, the uniform-flow condition is frequently assumed in the computation of flow in natural streams. The results obtained from this assumption are understood to be approximate and general, but they offer a relatively simple and satisfactory solution to many practical problems.

Not only should stream team members understand that Mannings equation provides a uniform flow approximation, they should also know the kinds of natural conditions that cause the deviation from reality to be either small or large. For this understanding, one should look not at a single cross-section, but at a stream reach. And the same factors that affect boundary resistance to the flow, or roughness, affect how closely the actual stream flow may be approximated by normal depth.

In Mannings equation, resistance to the flow is accounted for by the n-value, known as Mannings roughness coefficient. This empirical coefficient is not easy to correctly assertain by observation, as it is dependent on many physical aspects, including streambed composition, vegetation, cross-section irregularity (in the reach), channel alignment (straight or meandering), obstructions in the flow, and
sediment transport. In addition, the $n$-value for the same cross-section may vary significantly with depth and discharge.

Chow (1959) offers procedures for estimating Mannings $n$-values for channels and overbanks. Another frequently used procedure is provided by the US Geological Survey in Arcement and Schneider (1989). The more physical variation a stream reach exhibits the less likely it will flow at normal depth, and the more likely that it will be experiencing backwater. This term refers to what hydraulic engineers call a subcritical flow regime, in which roughness conditions toward the downstream end of a reach control the depth, and a water surface deeper than normal depth is backing up through the reach. In mildly sloped streams backwater conditions are very often present. A stream flowing at normal depth is experiencing no backwater (and Mannings equation cannot account for it). However, a lack of backwater does not mean the flow condition is not subcritical. Normal depth can occur in either subcritical or supercritical flow regimes.

A key tip for stream team members who make field observations is to note the variability of those flow resistance factors in the stream reach. For example, if two channel cross-section in the reach have different widths or shapes or have significantly different vegetation, the reach will probably not experience uniform flow. And if the channel bottom slope varies in the reach, which slope should be assumed for Mannings equation? Knowing that the equation will compute a depth that only approximates the real condition, the user may wish to make several calculations with different slopes to see how much the calculated depth changes with that factor. The Mannings roughness value is known to be a very significant variable in the equation. A prudent stream team member may wish to calculate how much the Mannings equation output changes with different estimations of $n$-value.

Since Mannings equation is applied at a single cross-section, it is important to understand the assumptions made for hydraulic parameters at that one location. For example, anyone who has observed a natural stream will notice that the flow is faster in the deeper middle part of the cross-section, and less fast near the edges where vegetation may be impeding the flow. But Mannings equation contains only one variable for velocity, which is the average velocity of the entire cross-section. If the roughness varies in different parts of a cross-section, how does the practitioner determine “composite” $n$-values for Mannings equation?

**Example calculations**

A simple cross-section will be used to show how the hydraulic parameters for Mannings equation are determined, and the output of three existing software programs will be compared. The cross-section (below) shows a water surface at elevation 13.5, and the magenta triangles and rectangles show subdivisions of flow area up to elevation 13. Four roughness coefficients have been estimated for the
To determine the uniform flow of this cross-section we may use one of the forms of Mannings equation. Note that both require cross-sectional flow area. (Although the velocity form does not contain the variable $A$, the hydraulic radius must be computed from $A$ divided by wetted perimeter.)

Consider the flow level at elevation 13 for slightly easier computations. One can divide the flow area into little triangles and rectangles (as shown) and compute the total by hand. Wetted perimeter is the length of the brown cross-section lines below elevation 13. The channel slope, $S$, must be a given— for this example, assume a slope of 0.005 feet vertically per foot of channel.

What about a composite $n$-value? Chow (1959, page 136) gives two formulas, explaining the assumptions behind each. (The two methods do not result in wildly different composite $n$-value estimations.) Both equations make use of the wetted perimeter lengths associated with each $n$-value in the cross-section. Thus, for the cross-section above, four wetted perimeter sub-values are used, corresponding to the four roughnesses present along the flow boundary.

At right, the first equation (6-17 in Chow) is based on the assumption that each of the

$$n = \left(\frac{P_{1}n_{1}^{1.5} + P_{2}n_{2}^{1.5} + P_{3}n_{3}^{1.5} + P_{4}n_{4}^{1.5}}{P_{total}}\right)^{2/3}$$

is different parts of the cross-section, with, for example, and $n$-value of 0.05 for the left bank. Within the channel, higher $n$-values on the banks indicate heavy vegetation. Note the section of vertical bank on the right side of the channel. Also, the lower roughness value on the right bank is due to the effect of cattle grazing.
four sub-areas of flow has the same mean velocity, equal to the mean velocity of the entire cross-section. \( P \) is wetted perimeter.

The second equation Chow gives (6-18) is based on the assumption that the total force resisting the flow in the cross-section is equal to the sum of the forces resisting the flow in each of the subdivided areas.

\[
n = \left( \frac{P_{1}n_{1}^{2} + P_{2}n_{2}^{2} + P_{3}n_{3}^{2} + P_{4}n_{4}^{2}}{P_{\text{total}}^{1/2}} \right)^{1/2}
\]

For the above cross-section, with water surface at elevation 13, the table at left shows the calculation of composite \( n \)-value by using the two equations. A different cross-section with different roughness combinations could, of course, show a wider difference, but this shows that the difference is not an order of magnitude.

The uniform flow calculations using Mannings equation and the above hypothetical cross-section were computed using three software applications, HecRAS, WinXSPRO, and the NRCS spreadsheet “Cross-Section Hydraulic Analyzer”. HecRas, from the US Army Corps of Engineers (Brunner, 2008) has a Uniform Flow calculator in its Hydraulic Design menu. The computations in HecRAS were made for water-surface elevation 13.5 and a screenshot of HecRAS results are shown below:

The same calculation, using the NRCS spreadsheet (Moore, 2009) is shown below. The spreadsheet

The red box from the screenshot above is enlarged here:

Note that by inserting the water surface elevation of 13.5 in the box near the bottom of the window, and pressing Compute, near the top above the cross-section plot, the computed discharge is 1305 cfs. One may calculate water surface elevation by blanking out that box, entering a discharge and pressing Compute. Or obtain discharge by blanking out the discharge box and entering water surface elevation.
computes a rating curve for each half-foot level of the cross-section, but also allows the user to compute parameters for any specific water surface elevation. The entire rating curve is shown, with an inset of the output for elevation 13.5 enlarged for better readability.

Note that the calculated discharge is 1336 cfs. This is slightly higher than the HecRAS value of 1305 cfs. How is this difference be explained? The HecRAS Hydraulic Reference Manual documents that the program computes composite n-value using Chow’s equation 6-17, whereas the NRCS spreadsheet uses Chow’s equation 6-18. But a closer look at HecRAS reveals a possible internal inconsistency in the program. (However, it should be noted that computing Mannings equation for a single cross-section, such as is done with the HecRAS Hydraulic Design calculator, can reasonably employ different assumptions than those of a Bernoulli equation approach between two or more cross-sections, as the program does in its steady flow analyses.)

A similar output table from HecRAS can be used to evaluate the spreadsheet rating curve.
(The highlighted line in the table is enlarged below.) This output table has been produced by the HecRAS steady flow analysis, rather than the HecRAS Hydraulic Design Uniform Flow calculator. Note that a user can perform uniform flow computations using the steady flow analysis capability of HecRAS as long as the cross-sections are identical and slope profile constant. (Of course, the roughness must also not change between the two cross-sections.)

The upstream cross-section 600, in HecRAS table, is identical to the one used in the NRCS spreadsheet. Another important step is that the user must set the HecRAS downstream flow boundary to be “normal depth”. The 100-foot reach has two cross-sections, and the downstream one has been lowered 0.5 feet to obtain the required 0.005 slope. By running a range of flows set identical to the spreadsheet output, HecRAS computes the output table below. The row for elevation 13.5 has been highlighted and the comparable section from the NRCS spreadsheet follows:

The HecRAS steady flow output shows an energy grade line slope of 0.005328, so it is not quite computing “normal depth”. (The slope would have to be exactly 0.005.) But notice that the flow of 1335.87 corresponds to a water surface of 13.51. The NRCS spreadsheet goes the other way, taking the depth and computing the discharge. But the pair of values are elevation 13.5 and flow 1335.87.

This evaluation seems to show that the steady flow analysis capability of RAS does a better job of computing uniform flow in a cross-section than does its Hydraulic Design function. (The explanation probably involves a difference in the HecRAS composite roughness computation between the two ways of performing the analysis.)

Users may get differing results for uniform flow for another reason. Remember that Mannings equation uses average velocity for the entire cross-section. Certain cross-section shapes and horizontal changes in roughness can throw a curve-ball at the user. If total cross-section Mannings results are compared with those of a sub-divided cross-section, the difference can be significant.
When flow breaks over a bank, especially if the overbank is relatively wide and flat, Mannings equation applied to the entire flow area may not do a good job of estimating the true uniform flow. This will be further explained below. The overall average velocity may be too affected by relatively high overbank roughness. (Consider another issue: at the stage shown in the photograph above, and for similar stream reaches, the likelihood of the discharge being uniform flow is quite remote.)

The third program in this uniform flow analysis comparison is the Forest Service application WinXSPro (Hardy, 2005). The same cross-section is input and the resulting rating curve table is shown below. This program does not compute the overall flow using the average values of the entire cross-section. It divides the section into a different part for each different roughness value, computes the discharge component of each and sums them for the total.

The rating curve is upside-down, compared to the NRCS spreadsheet, and is given relative to stage, or depth from the thalweg, rather than in elevation. Thus, elevation 13.5 in the NRCS spreadsheet corresponds to stage 6.5 in WinXSPro. The “T” in the highlighted line above indicates the WinXSPro computations for the total cross-section. Note the flow in the Q column of 1832.65 cfs. This is much higher than the NRCS and HecRAS estimates of 1335.87 cfs. Why the difference? The Forest Service program does not compute composite roughness values. It computes a separate discharge for each subarea of a given \( n \)-value and sums them up. In the table the four subareas are labeled A, B, C, and D. Subarea B represents the short vertical section of the right bank and WinXSPro does not count that wetted perimeter roughness at all. (The flow area at stage 6.5, subarea B, is zero.) Nevertheless, when the program sums up all the subareas it still overshoots the correct value.

As explained above, Mannings equation may not make a very good estimate when overbank flow is wide and shallow, and subdividing the overbanks from the channel would be an improvement. But WinXSPro subdivides under all conditions. This is not a good practice, because it is equivalent to assuming the various subareas of flow are separated from each other by glass walls. Real roughness elements do not have this restriction. By using a composite roughness, a more physically correct analysis is possible. The effect of flow turbulence is not restricted within vertical columns but causes eddys both horizontally and vertically in the flow, which increases flow resistance.

Thus, WinXSPro over-estimates the discharge in this cross-section. The user manual, page 16 (Hardy, 2005) explains that,
WinXSPRO assumes frictionless vertical divisions (‘smooth glass walls’) between individual subsections. This assumption of negligible shear between subsections avoids the formidable task of estimating small energy losses due to friction and momentum exchanges between adjacent moving bodies of water.

In reality, the physics of flow is very complicated— no doubt about it. But WinXSPRO should have handled the “formidable task” by computing composite roughness values.

Practitioners should keep in mind that Mannings Equation produces cross-section averaged hydraulic parameters. Even more sophisticated procedures such as step-backwater water surface profile computations (used by HecRAS steady flow) can only approximate the subdivision of hydraulic parameter values within a cross-section. When using Mannings equation, the best procedure would be to subdivide flow sections only when flow in the overbanks is significantly wider and rougher than the channel.

Both the NRCS spreadsheet and HecRAS steady flow modeling compute composite roughness. In addition, within the channel they test whether the flow should be subdivided for a better estimate. The NRCS spreadsheet test is looking for a significant enough side-slope break into the overbank. (In other words, whether the overbank suddenly widens and flattens with increasing depth in the cross-section.) The HecRAS Hydraulic Reference Manual explains the testing of that model, stating:

Flow in the main channel is not subdivided, except when the roughness coefficient is changed within the channel area. HEC-RAS tests the applicability of subdivision of roughness within the main channel portion of a cross-section, and if it is not applicable, the program will compute a single composite n value for the entire main channel.

Using the NRCS spreadsheet with the above cross-section, one can see the effect of subdividing the flow for the left overbank. For elevation 14 the hydraulic parameters are the following:

The little red triangles in the upper right cell corners indicate comments that show the user how the data was subdivided between channel and overbanks. But if one takes the totals into the velocity form of Mannings equation, the result is less than the 5.48 feet per second shown above.

\[
V = \frac{1.486}{n} \times R^{2/3}S^{1/2} = \frac{1.486}{0.0439} \times (2.681)^{2/3} (0.005)^{1/2} = 4.619 \text{ feet per second}
\]

Multiplying by the flow area, the resulting discharge would be 1366 cubic feet per second. In the above screenshot, the discharge is given as 1621 cfs. Without subdividing the flow in the left overbank, in this case, the overall discharge would be underestimated by 255 cfs. The following screenshot shows Excel comments in data cells which provide the user details on the subdivided data.

The spreadsheet computes the channel velocity and left overbank velocity separately, then obtains the
channel discharge (1573.1 cfs) and the left overbank discharge (48.1 cfs). The sum of these two sub-
discharges is the better approximation of normal depth in the cross-section (the 1621 cfs shown). Note
that this example shows the significant benefit of floodplain connectivity for streams. The flow
velocity is significantly lower and less damaging in the rough overbank.

Summary

The physics of flowing water, or fluid mechanics, is a subject of study in civil engineering curricula.
Even though many projects involving streams do not need advanced physical analysis, approximate
methods should be used with care. Interdisciplinary teams working on stream projects often include
biologists and economists who are not likely to have studied hydraulic engineering as part of their
education. The concepts involved in the use of Mannings equation are not difficult, but a good
understanding does require some study and attention to detail. Even engineers may sometimes forget
the appropriate scope for application of technical equations and procedures.

The best advice to apply when determining the adequacy of a Mannings equation estimate would be to
examine physical changes in the reach for the flow stages of interest. The greater the extent that non-
uniformity is observed, the further from reality a uniform flow estimate may expected to diverge. At
least one can in most cases be sure that the equation will overestimate the flow for a given stage
because any non-uniformities in the reach are likely to produce backwater.

The NRCS spreadsheet may be downloaded here:
  xsecAnalyzer (web page)  or  xsecAnalyzer (Excel file)

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